FTIR and the illusion of superluminality

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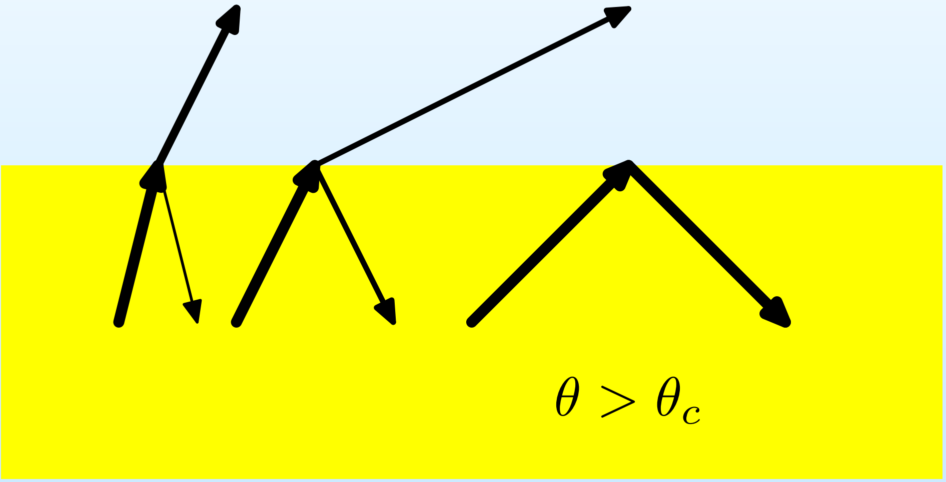
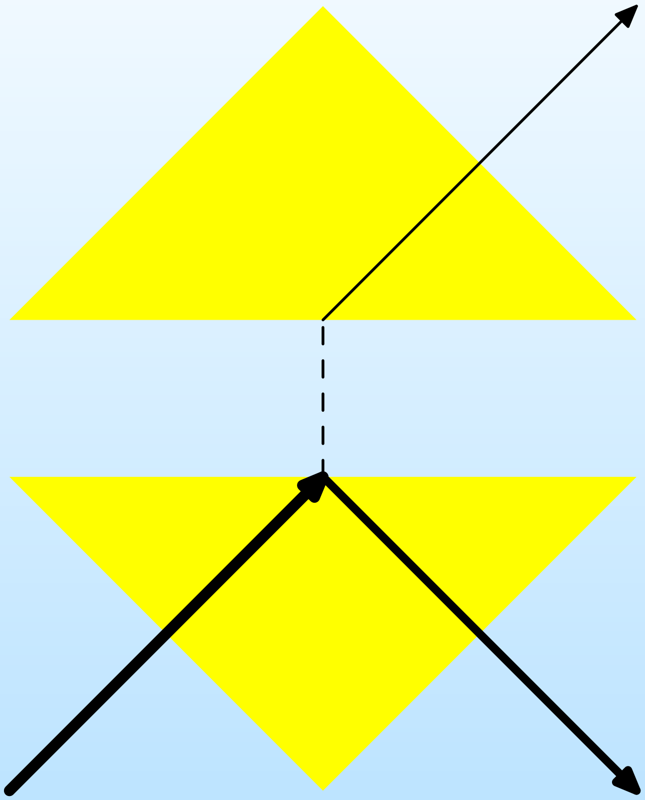
Total Reflection



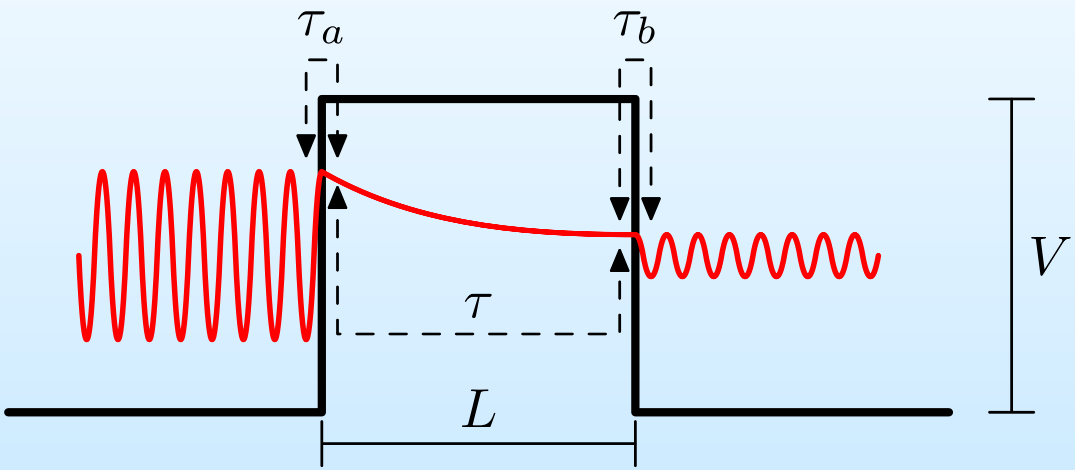
FTIR



Time to cross air gap $\tau =$?

Tunneling through barriers

If $E < V$



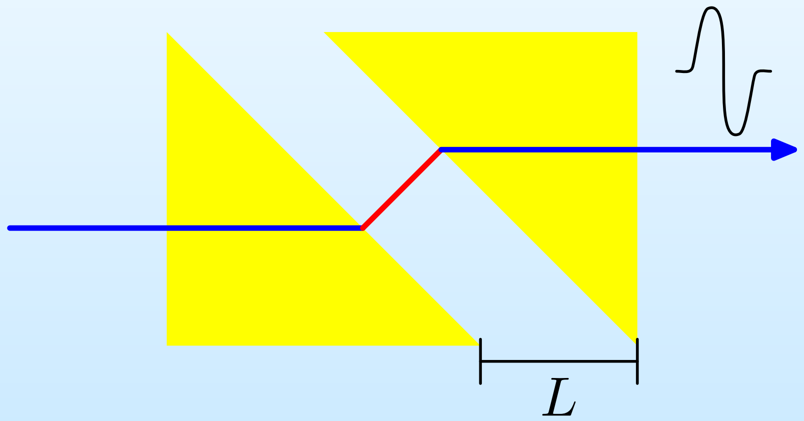
Phase	$\phi = 0$
Crossing time	$\tau = \frac{\partial \phi}{\partial \omega} = 0$
Speed	∞

Similarly in FTIR, the first crossing time $\tau = 0$.

- Schrödinger's equation is *non-relativistic*. . .
- However, Maxwell's equations are relativistic; light propagation should comply with Einstein's causality!

Experiment

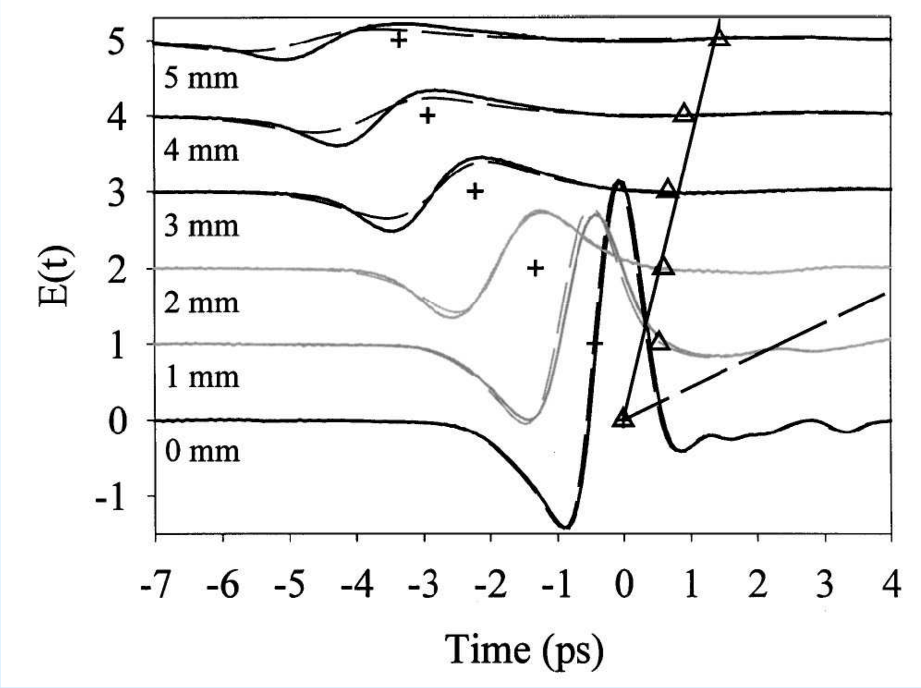
Carey, Zawadska, Jaroszynski y Wynne, PRL **84**, 1431 (2000)



$\lambda \approx 1$mm
$\Delta x \approx 2$mm

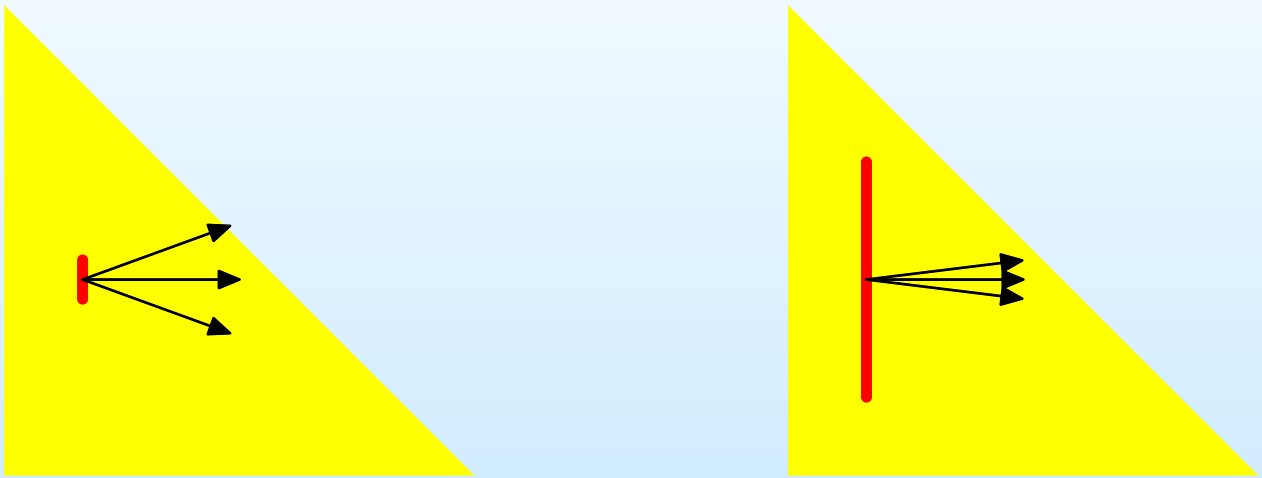
Measured $E(t)$ as L varies. . .

Experiment



- Peak advance larger than width.
- Superluminal and causality violating propagation.
- $\tau \approx 0$.

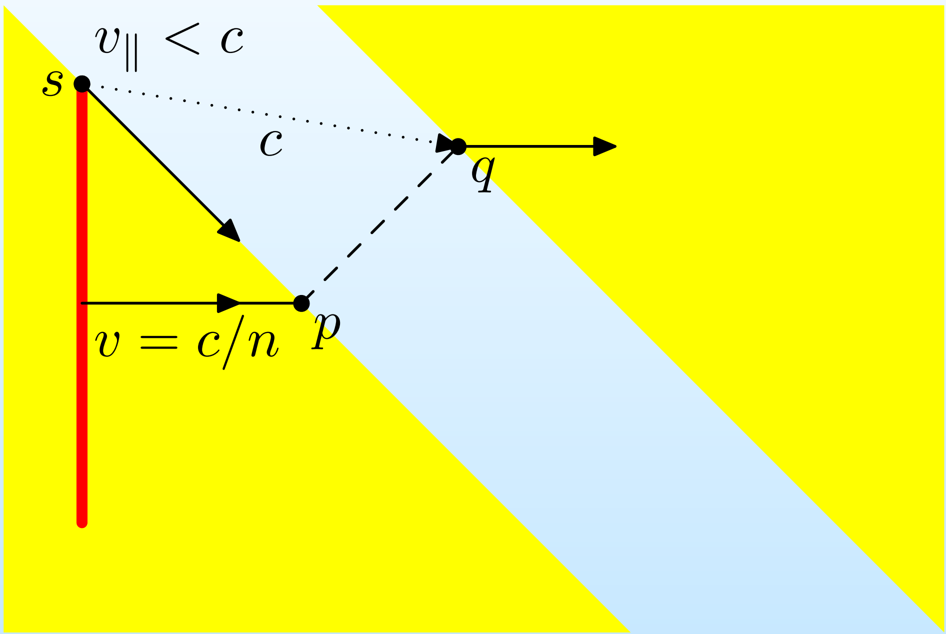
Uncertainty Relations



$\Delta\theta\Delta x \approx \lambda$

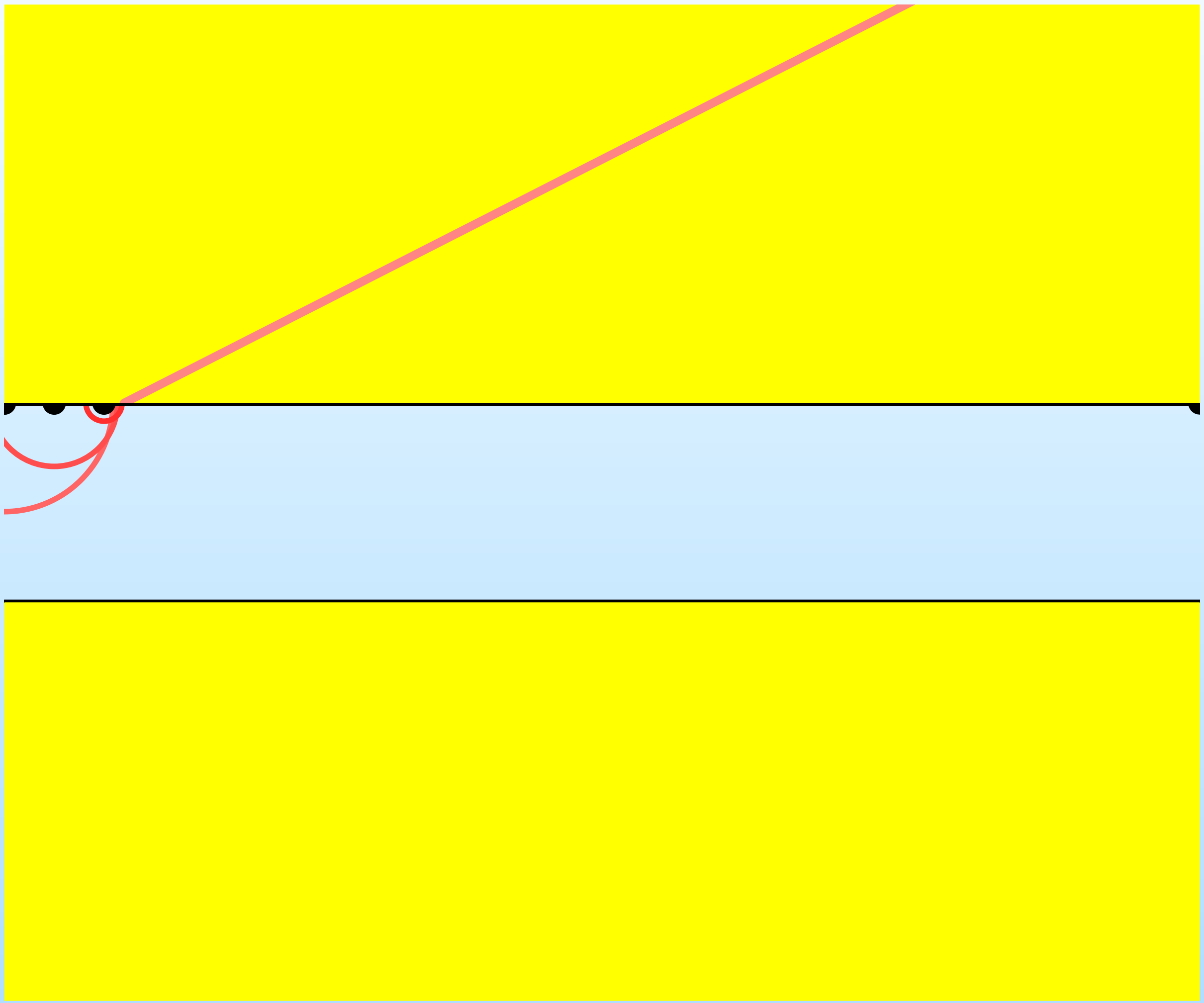
$\theta > \theta_c$ requires laterally extended wavefronts.

Velocity along surface



- If $\theta > \theta_c$, point s moves *slowly*, at $v_\parallel = c/(n \sin\theta) < c$, towards p.
- There might be enough time to propagate at speed c from s to q in order to arrive at the same time as the wavefront arrives at p.

Huygens construction $\theta < \theta_c$



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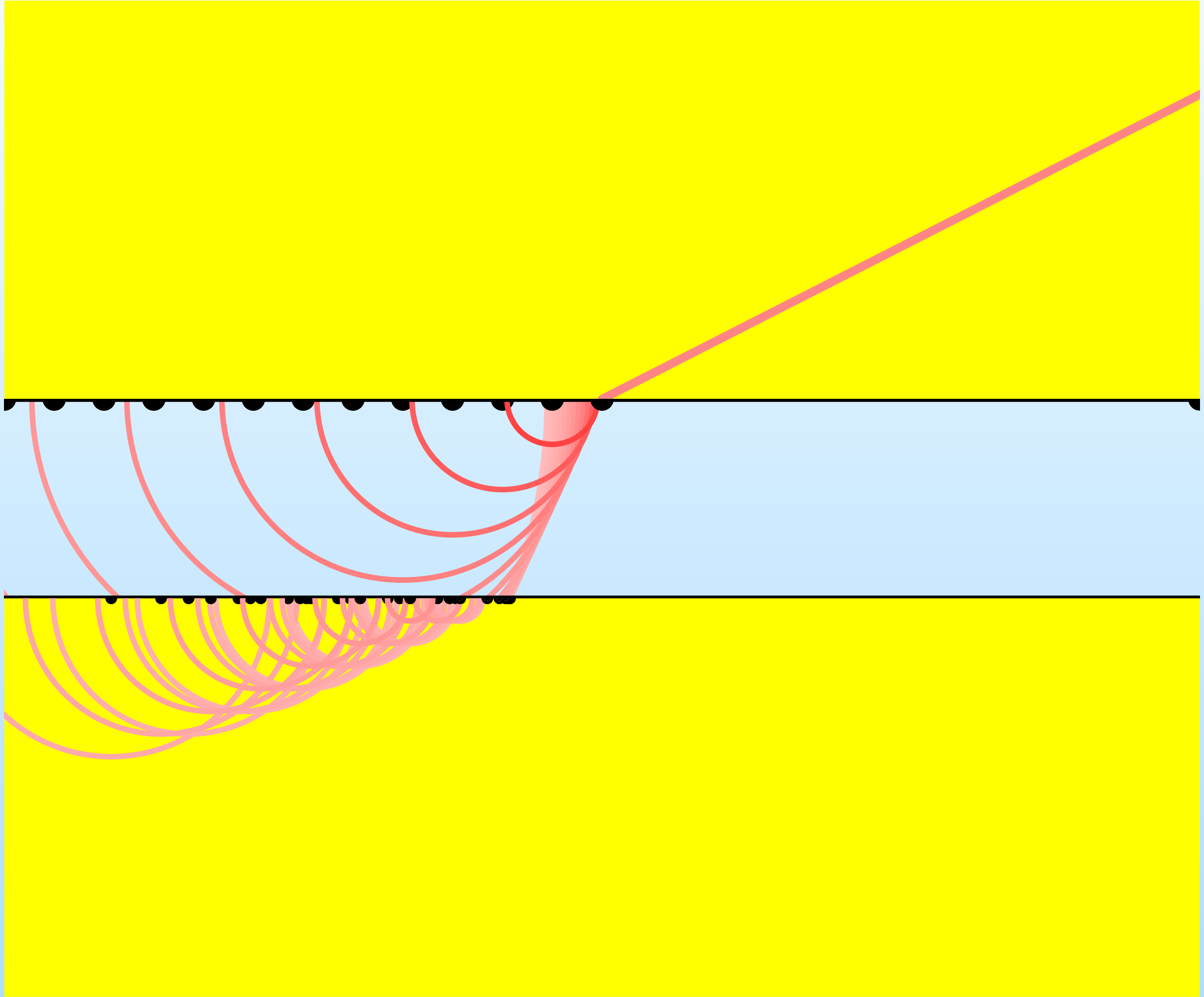

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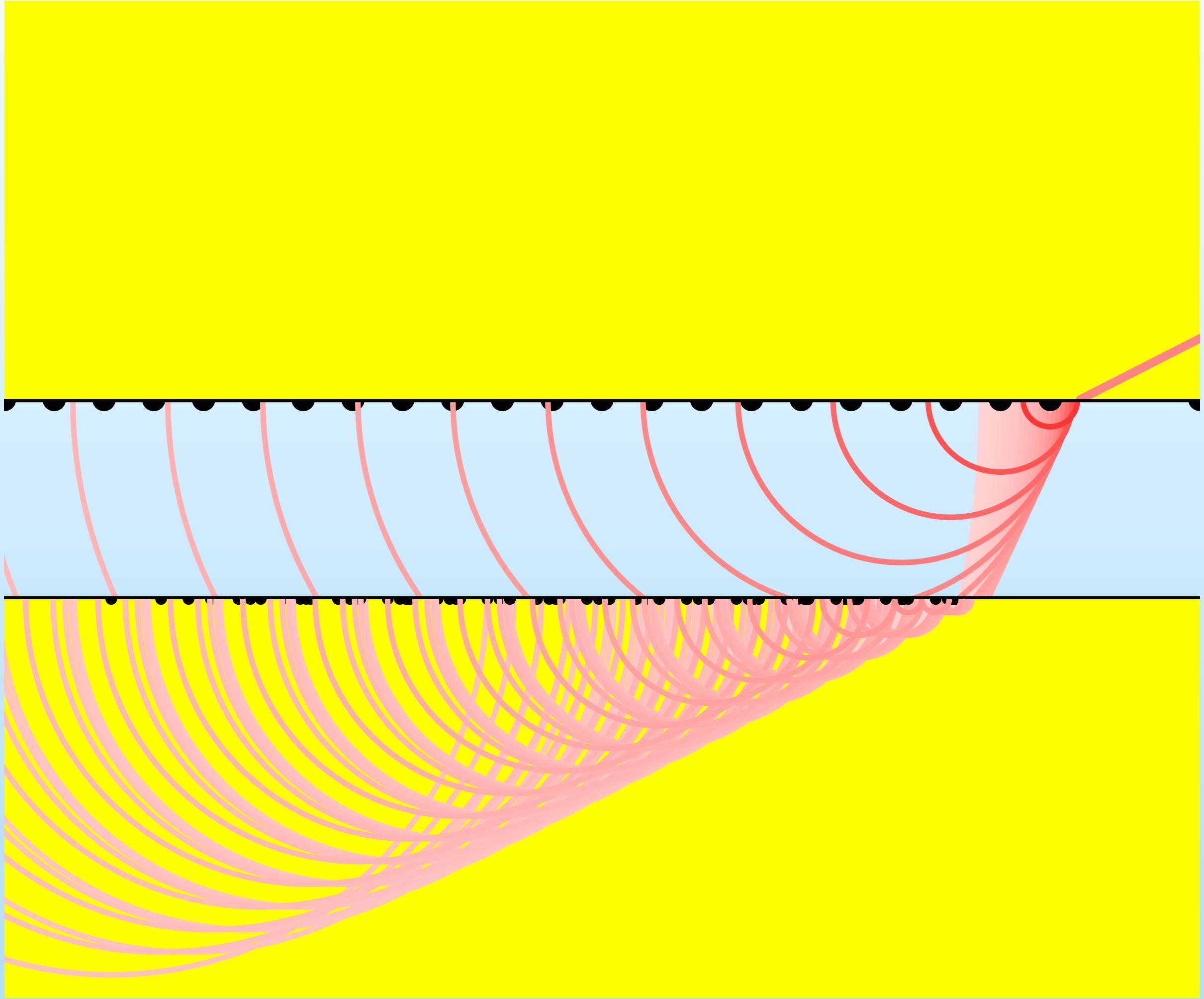

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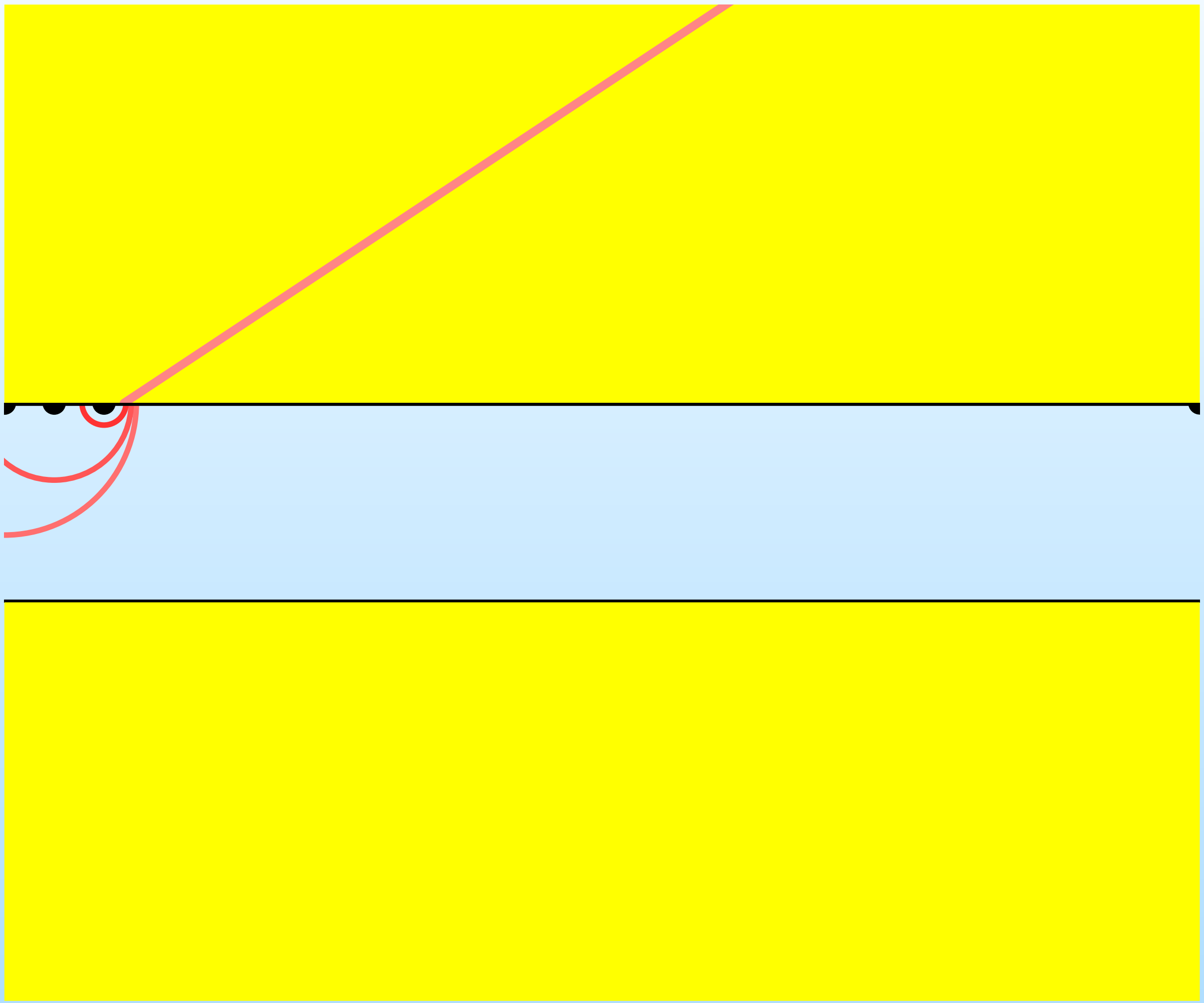

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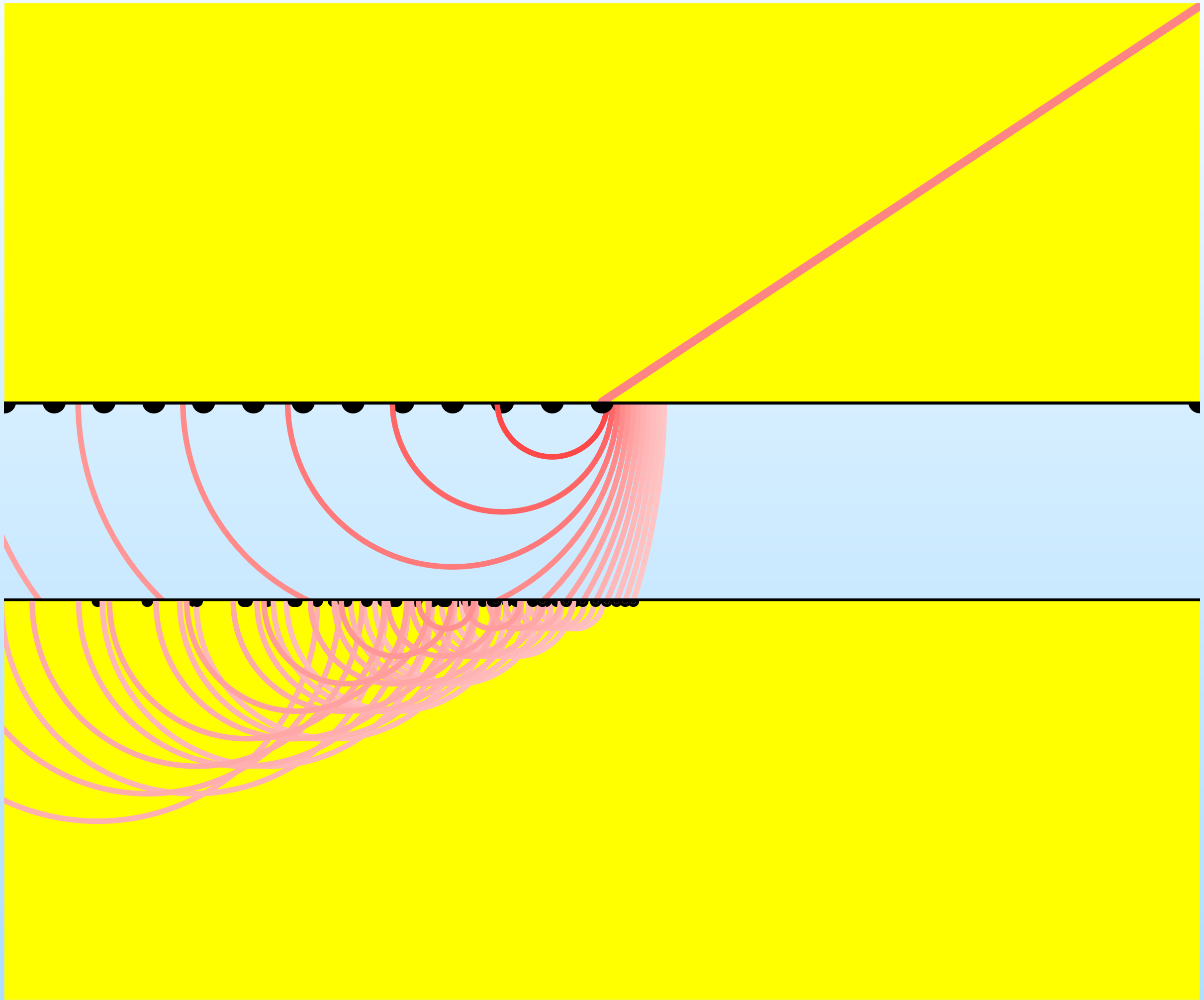

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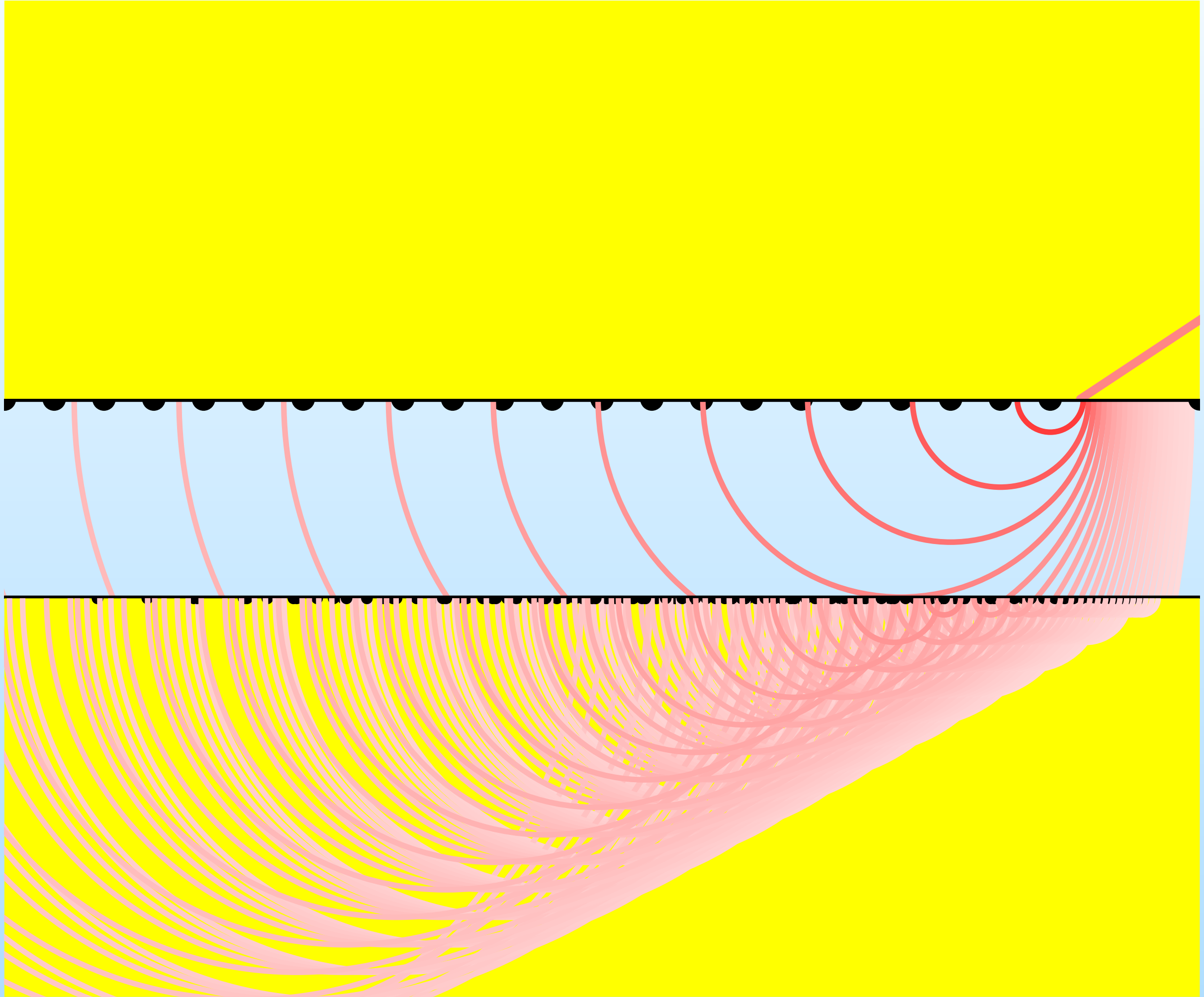


Retarded Causal Propagator

- Free space 2D Green's function

$$G_0(t,x,z;t',x',z') = \frac{c}{2\pi} \frac{\Theta\left(c(t-t') - \sqrt{(x-x')^2 + (z-z')^2}\right)}{\sqrt{c^2(t-t')^2 - (x-x')^2 - (z-z')^2}}$$

- Dirichlet Green's function for $z > 0$ semi-space (images)

$$G(t,x,z;t',x',z') = G_0(\ldots z - z') - G_0(\ldots z + z')$$

- Propagator from $(t',x',0) \rightarrow (t > t', x, z > 0)$ (Green)

$$P(t,x,z;t',x') = \frac{\partial}{\partial z'} G(t,x,z;t',x',z'=0)$$

$$\phi(t,x,z) = \int dx' \int dt'\, P(t,x,z;t',x')\phi(t',x',0)$$

Potential

- Ancillary field

$$\psi(t,x,z) = \frac{c}{\pi}\int dx' \int dt' \, \frac{\phi(t',x',0)}{\sqrt{c^2(t-t')^2 - (x-x')^2 - z^2}},$$

- from which

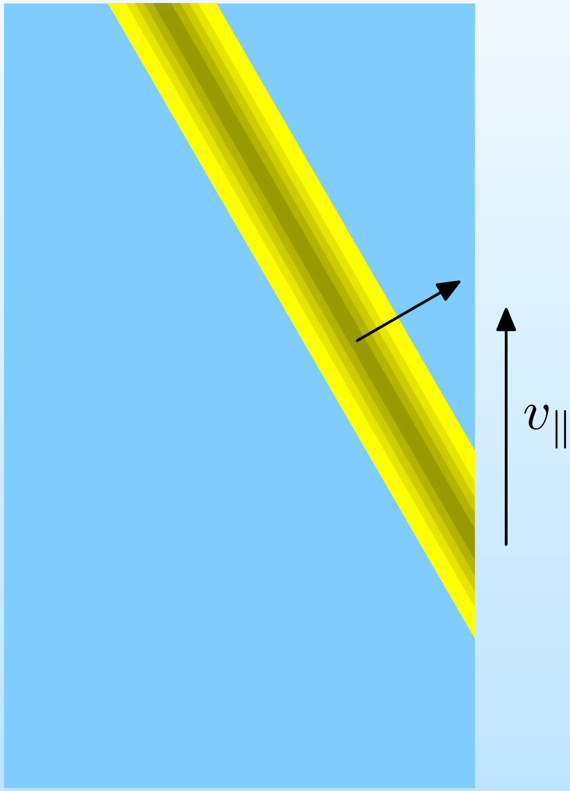
$$\phi(t,x,z) = -\frac{\partial}{\partial z}\psi(t,x,z).$$

- Integration region

$$c(t-t') > \sqrt{(x-x')^2 + z^2}$$

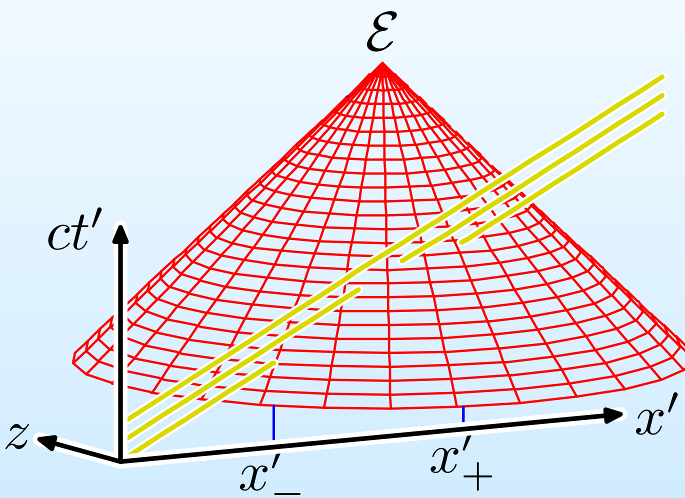
(causality).

Plane pulse



- $\phi(t, x, 0) = f(t - x/v_{\parallel})$
- $v_{\parallel} = c/(n \sin \theta)$
- $\theta < \theta_c \Rightarrow v_{\parallel} > c$
- $\theta > \theta_c \Rightarrow v_{\parallel} < c$

Non-evanescent case



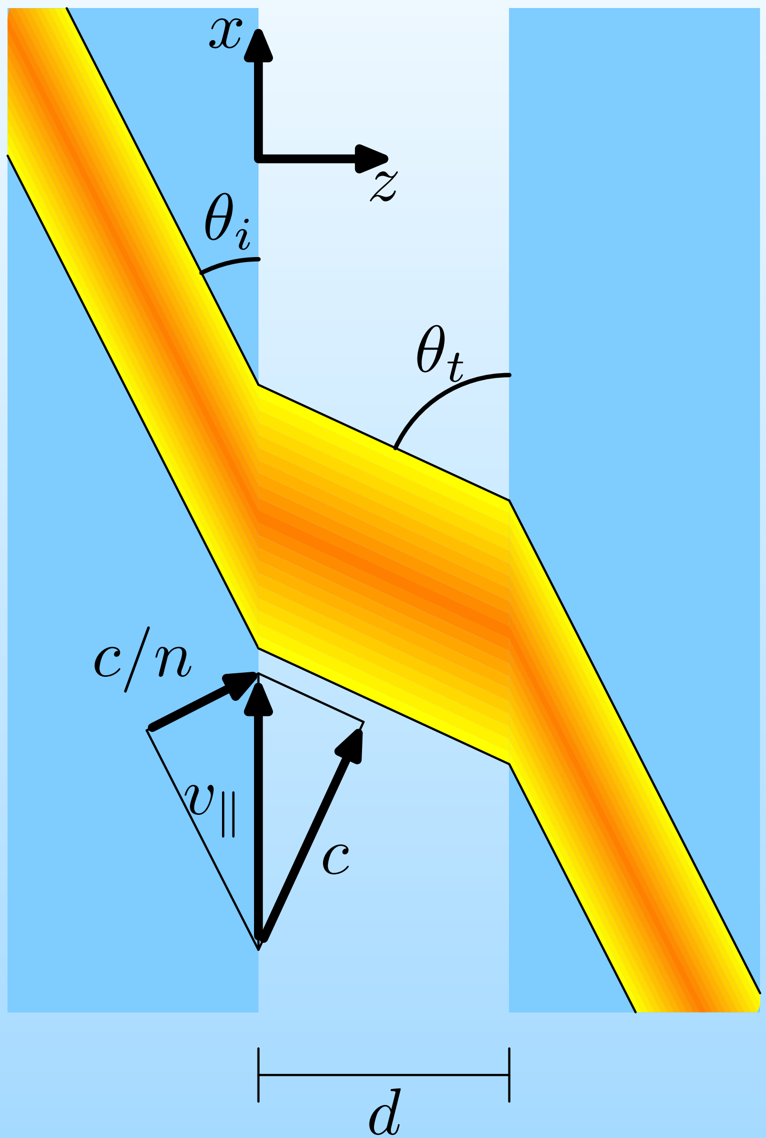
Bounds:
$t'' < t_m = t - x\sin\theta_t - z\cos\theta_t$,
$x_- < x' < x_+$

$$x' \to \eta \equiv \frac{x' - [x - c(t-t'')\sin\theta_t]}{\sqrt{[x\sin\theta_t - c(t-t'')]^2 - z^2\cos^2\theta_t}},$$

$$\psi(t,x,z) = \frac{c}{\pi\cos\theta_t}\int_{-\infty}^{t_m} dt''\, f(t'')\int_{-1}^{1}\frac{d\eta}{\sqrt{1-\eta^2}},$$

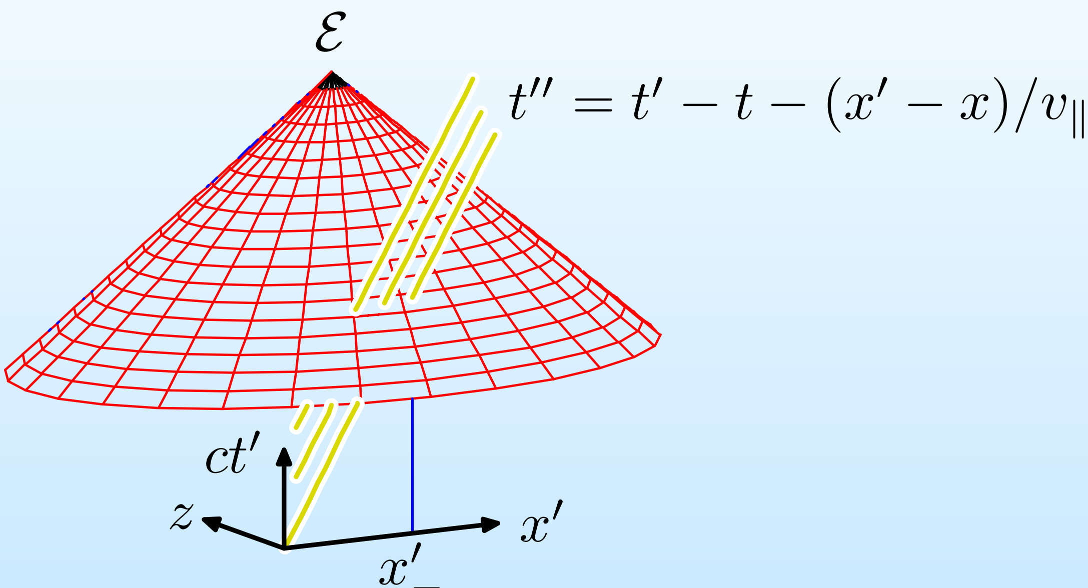
$$\phi(t,x,z) = f(t - (x\sin\theta_t + z\cos\theta_t)/c)$$

Non-evanescent pulse



- Transmitted pulse has the same profile as the incident pulse,
- propagating at speed c,
- with angle θ_t (Snell).

Evanescent case



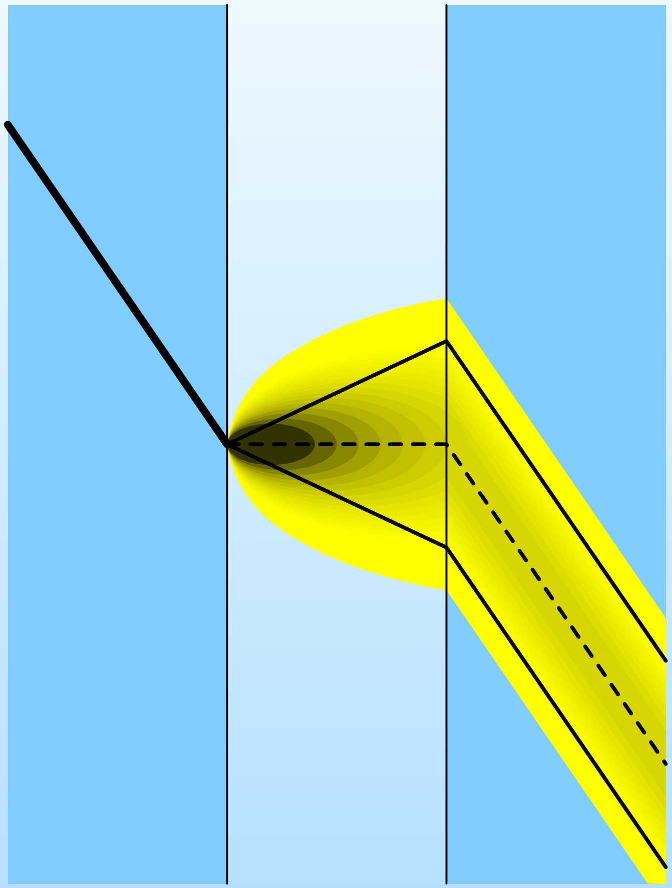
Bounds:
t'' unbounded,
$x' < x'_-$

$$x' \to \eta \equiv \frac{1}{\gamma\beta} \frac{x'' + \gamma^2 \beta c t''}{\sqrt{z^2 + (\gamma\beta c t'')^2}},$$

$$\psi(t, x, z) = \lim_{L \to -\infty} \frac{\gamma\beta c}{\pi} \int_{-\infty}^{\infty} dt''\, f_t(t - x/v_\parallel + t'') \int_{\eta_L}^{-1} \frac{d\eta}{\sqrt{\eta^2 - 1}},$$

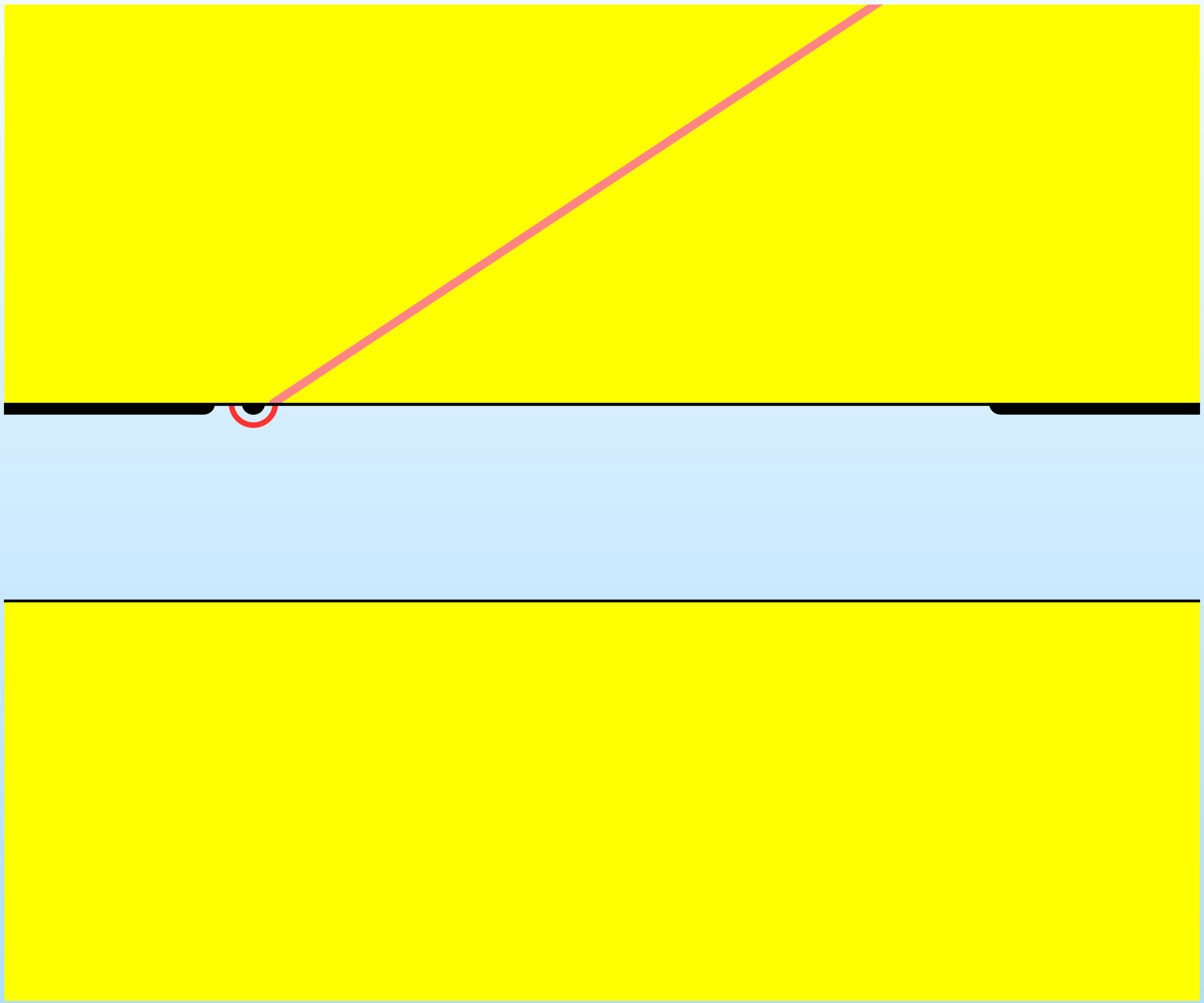
$$\phi(t, x, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} dt''\, f_t(t - x/v_\parallel + t'') \frac{\gamma |v_\parallel| z}{z^2 + (\gamma v_\parallel t'')^2}.$$

δ pulse

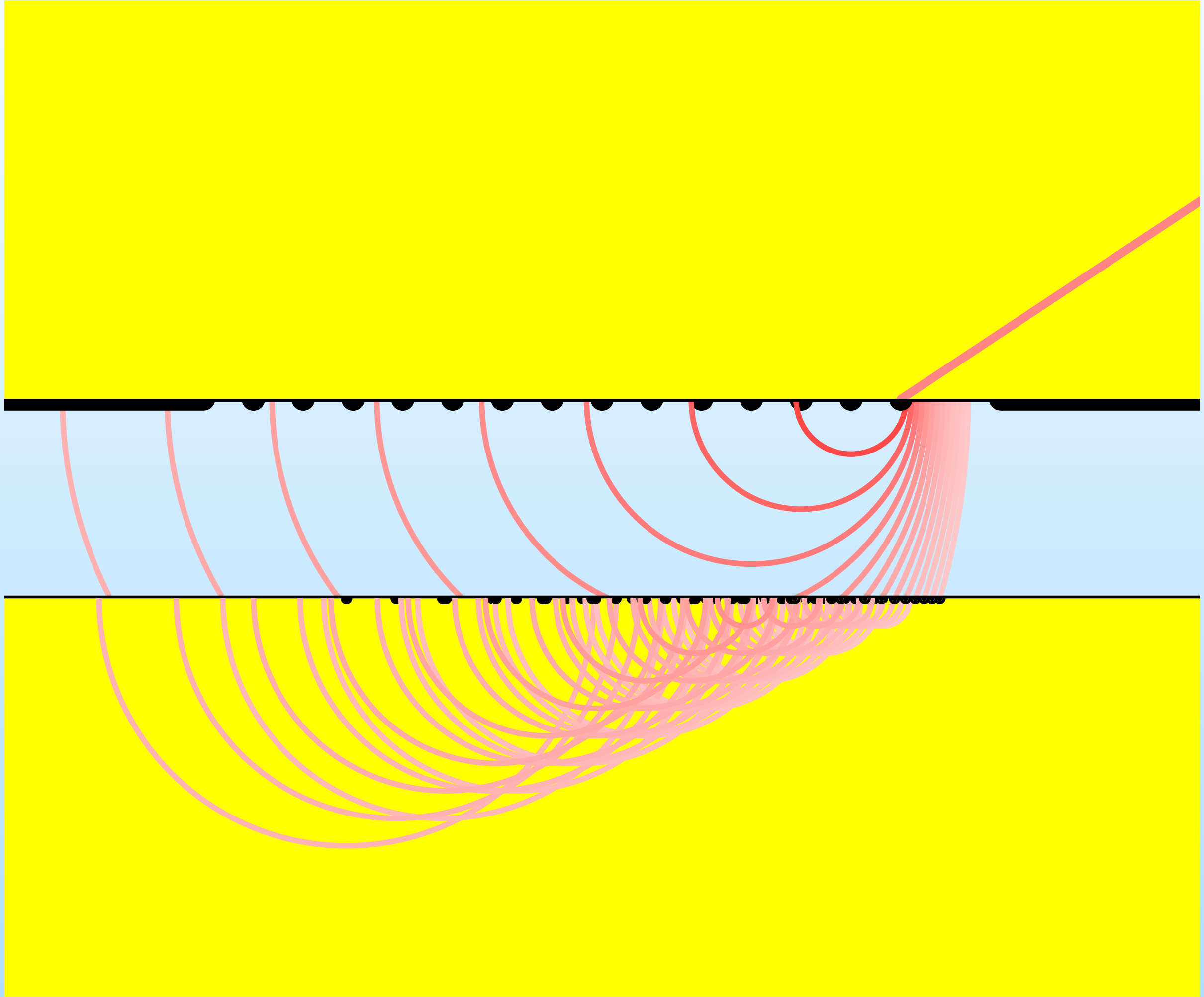


- $f(\tau) \equiv f_0 \delta(\tau) \Rightarrow$
- $\phi(t, x, z) =$
  $\dfrac{f_0}{\pi} \dfrac{\gamma |v_\parallel| z}{z^2 + \gamma^2 (x - v_\parallel t)^2}$
- Lorentzian of width z/γ centered on *actual* position of incident pulse on $z = 0$ surface.
- Power law, not exponential decay!
- Exponential decay restored for wavetrains due to interference.

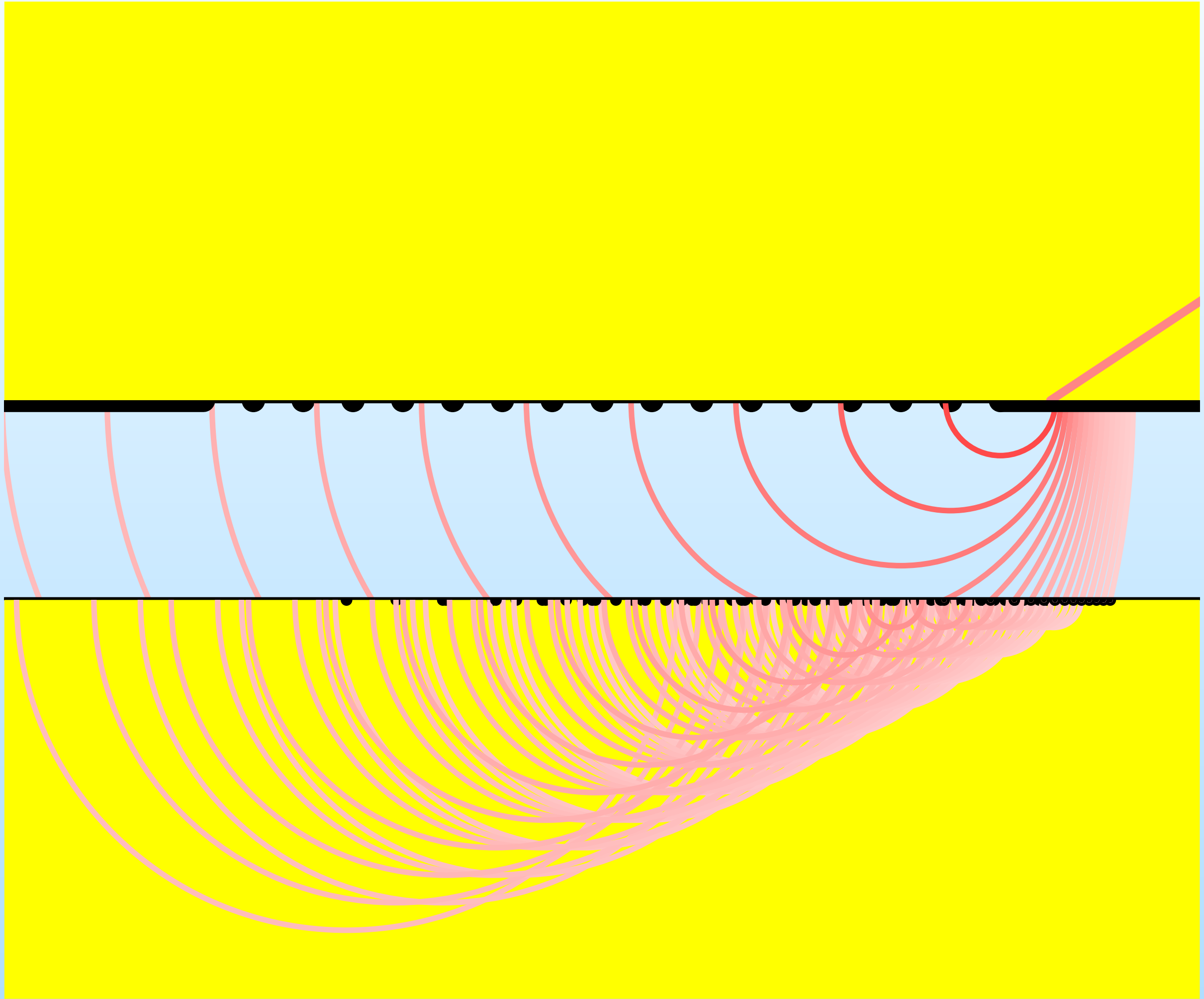
Opaque screens



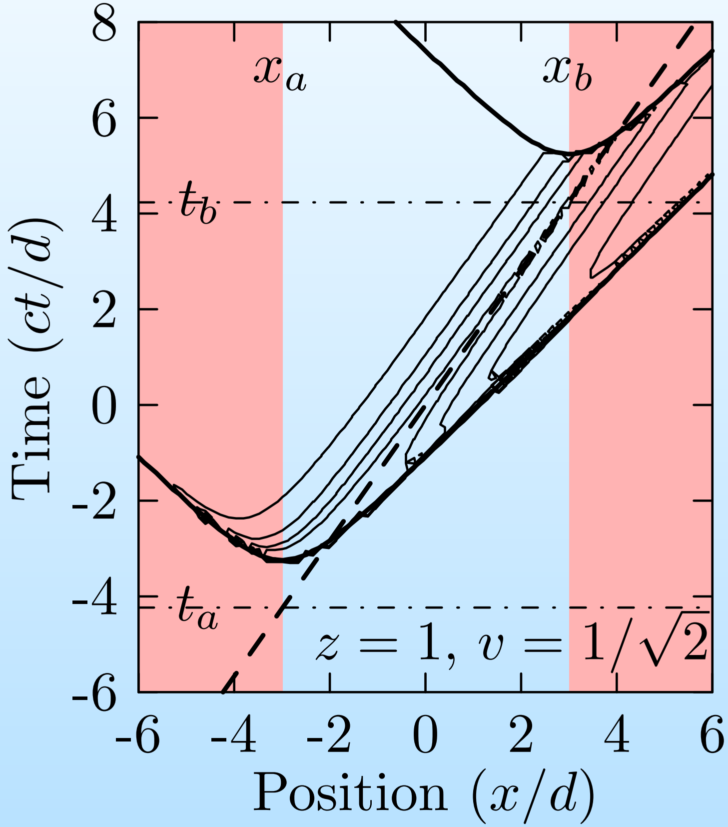
Opaque screens



Opaque screens



Test for superluminality



- Delay (ρ/c) from the uncovering time t_a to the field arrival.
- Delay (ρ/c) from the covering time t_b before the transmitted field *notices*.
- Field penetrates beyond screens.
- Diffraction singularity propagates with speed c.

$$\phi(t, x, z) = \frac{1}{\pi}\int_{-\infty}^{\infty} dt''\, f(t - x/v_{\parallel} + t'')\frac{\gamma v_{\parallel} z}{z^2 + (\gamma v_{\parallel} t'')^2}(1 - C).$$

$C =$ diffraction correction.

Problem

- Diffraction introduces non-evanescent components.
- How does a spatially localized (along both longitudinal and transverse directions) evanescent pulse propagates?
- Fourier superposition

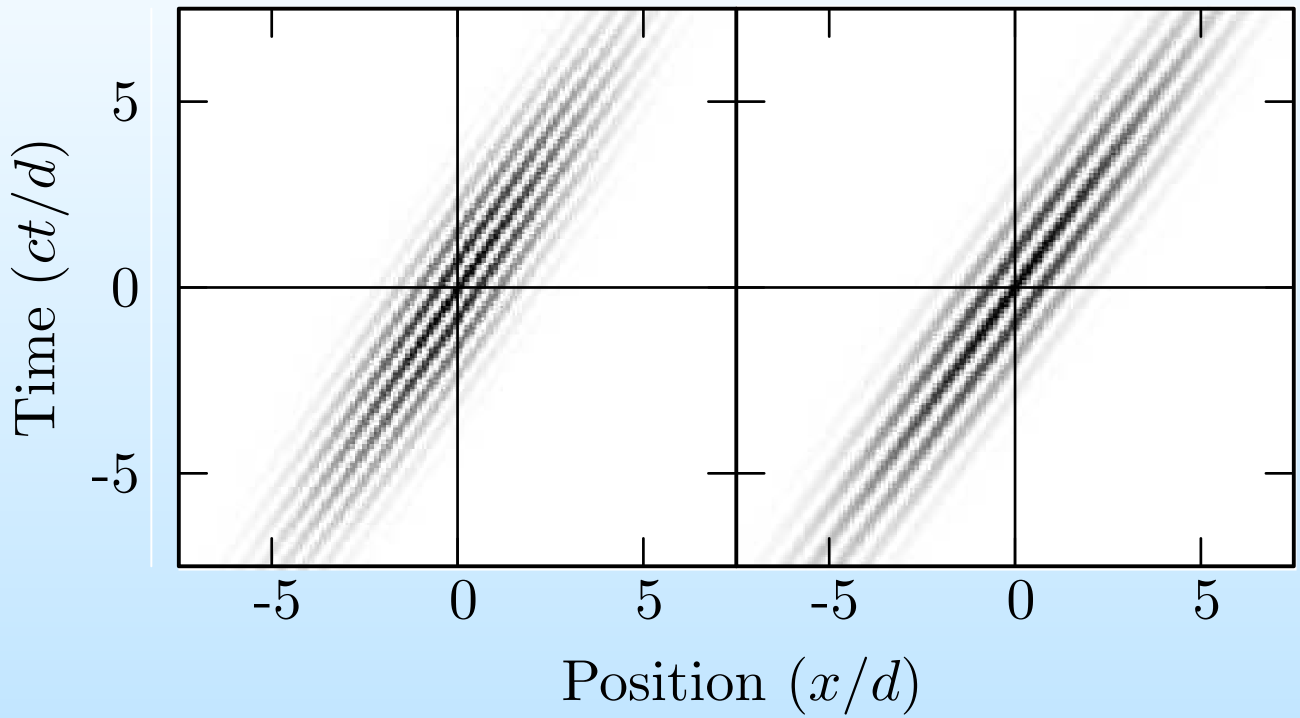
$$f_{\omega_Q} e^{[-i(\omega t - Qx) - \kappa z]}$$

with $\omega < |Q|$, or equivalently, through *velocity* superposition

$$f_{\omega v_\parallel} e^{-\omega[i(t - x/v_\parallel) - z/(\gamma v_\parallel)]}$$

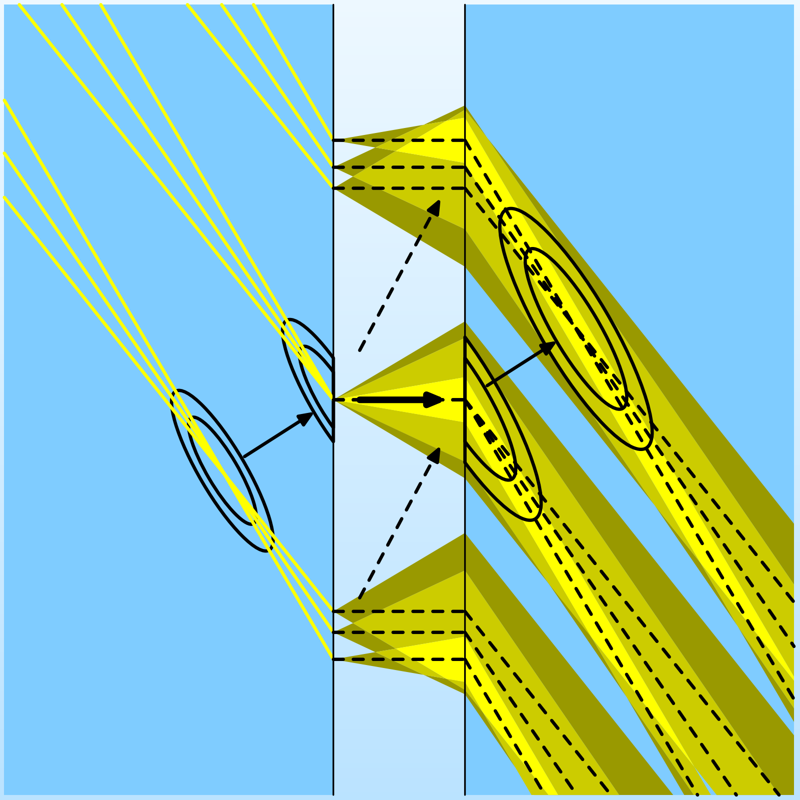
with $|v_\parallel| < c$.

Gaussian evanescent pulse



- $v_0 = 0.7c$, $\Delta v = 0.15c$, $\omega_0 = 16c/d$, $\Delta\omega = 2c/d$.
- Identical except for decrease in amplitude (10^{-12}), in frequency and in angle.

Propagation of localized pulses



- Pulse localized where wavefronts overlap.
- Slower wavefronts start ahead but finsih behind.
- Wavefront widen while crossing *causally*.
- Transmitted wavefronts overlap each other simultaneously with incident wavefronts, yielding an apparent superluminality.

Constrained fields

- Pulses made up of narrow directional components seem to propagate superluminally. Is this behavior generic?
- Fourier decomposition of evanescent fields,

$$\phi(t,x,z) = \int \frac{dQ}{2\pi} \int \frac{d\omega}{2\pi} e^{-i(\omega t - Qx) - \kappa z} \phi_{\omega,Q},$$

with $|\omega| < |Q|c$, $\kappa = \sqrt{Q^2 - \omega^2/c^2}$, and

$$\phi_{\omega,Q} = \int dt' \int dx' \phi(t', x', z' = 0) e^{-i(\omega t - Qx)}.$$

- Thus,

$$\phi(t,x,z) = \int dx' \int dt' \, P'(t,x,z;t',x') \phi(t',x',0)$$

Evanescent propagator

$$P'(t-t', x-x', z) = \int \frac{dQ}{2\pi} \int \frac{d\omega}{2\pi} e^{i[Q(x-x')-\omega(t-t')]-\kappa z}.$$

$$P' = \frac{c}{\pi^2} \begin{cases} [z \log|(z+s)/(z-s)| - 2s]/(2s^3) & \text{if } \rho > ct, \\ [\tau - z \arctan(\tau/z)]/\tau^3 & \text{if } \rho < ct, \end{cases}$$

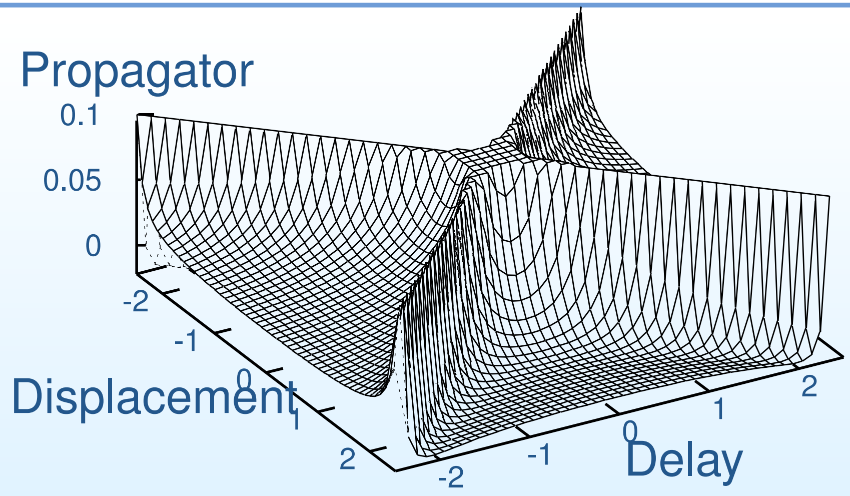
where

$$\rho = \sqrt{(x-x')^2 + z^2},$$

$$s \equiv \sqrt{\rho^2 - c^2(t-t')^2},$$

$$\tau \equiv \sqrt{c^2(t-t')^2 - \rho^2}.$$

Evanescent propagator

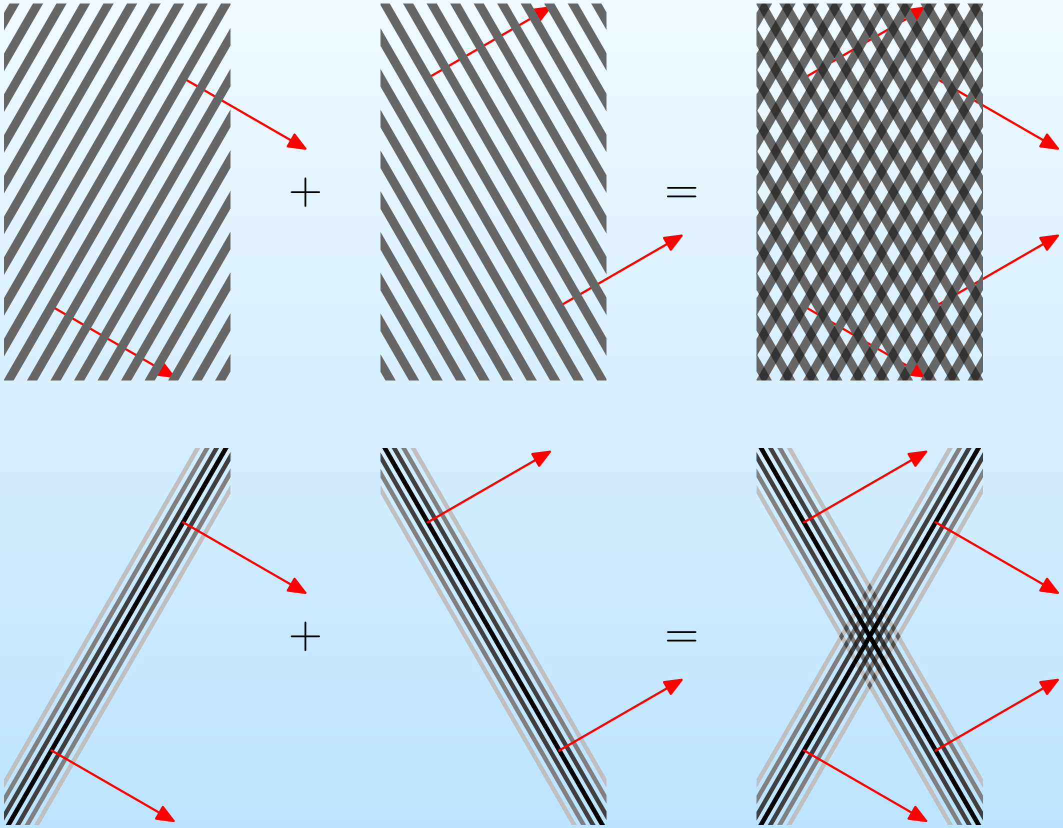


- $P'(t,x)$ is symmetric under $(x-x') \rightarrow -(x-x')$, $(t-t') \rightarrow -(t-t')$, thus, it is *superluminal and acausal!*.
- P' has singularities along $x - x' = \pm c(t-t')$
- The retarded, causal, (sub)luminal propagator P may be used for *arbitrary fields*.
- The acausal propagator P' yields the same results when acting on *evanescent fields*,
- making it *impossible* to distinguish superluminal from subluminal propagation of arbitrary evanescent pulses!

Predictability

- $\phi(t,x,z) = \int \frac{dQ}{2\pi} \int \frac{d\omega}{2\pi} e^{-i(\omega t - Qx) - \kappa z} \phi_{\omega,Q} = \int \frac{dQ}{2\pi} \phi_Q(t,z) e^{iQx}$.
- $\phi_Q(t,z)$ has finite spectrum $|\omega| < |Q|c$. Analytical with no singularities.
- Analytical continuation to arbitrary t from arbitrary neighborhood.
- $\phi(t \approx t_1, x, 0) \Rightarrow \phi(t \approx t_2 > t_1, x, 0)$. Subluminally transmitted $\phi(t \approx t_2, x, z)$ mimics $\phi(t \approx t_2, x, 0)$ giving the impression of superluminal transmission.
- Information and energy in the lateral wing of a localized pulse before arrival is enough to produce the complete transmitted pulse.
- Similar to the Coulomb field for charge in uniform motion.
- Failure when $Q \to \infty$. However, the decay length $\propto 1/Q$.
- Effects of thermal and quantum noise?

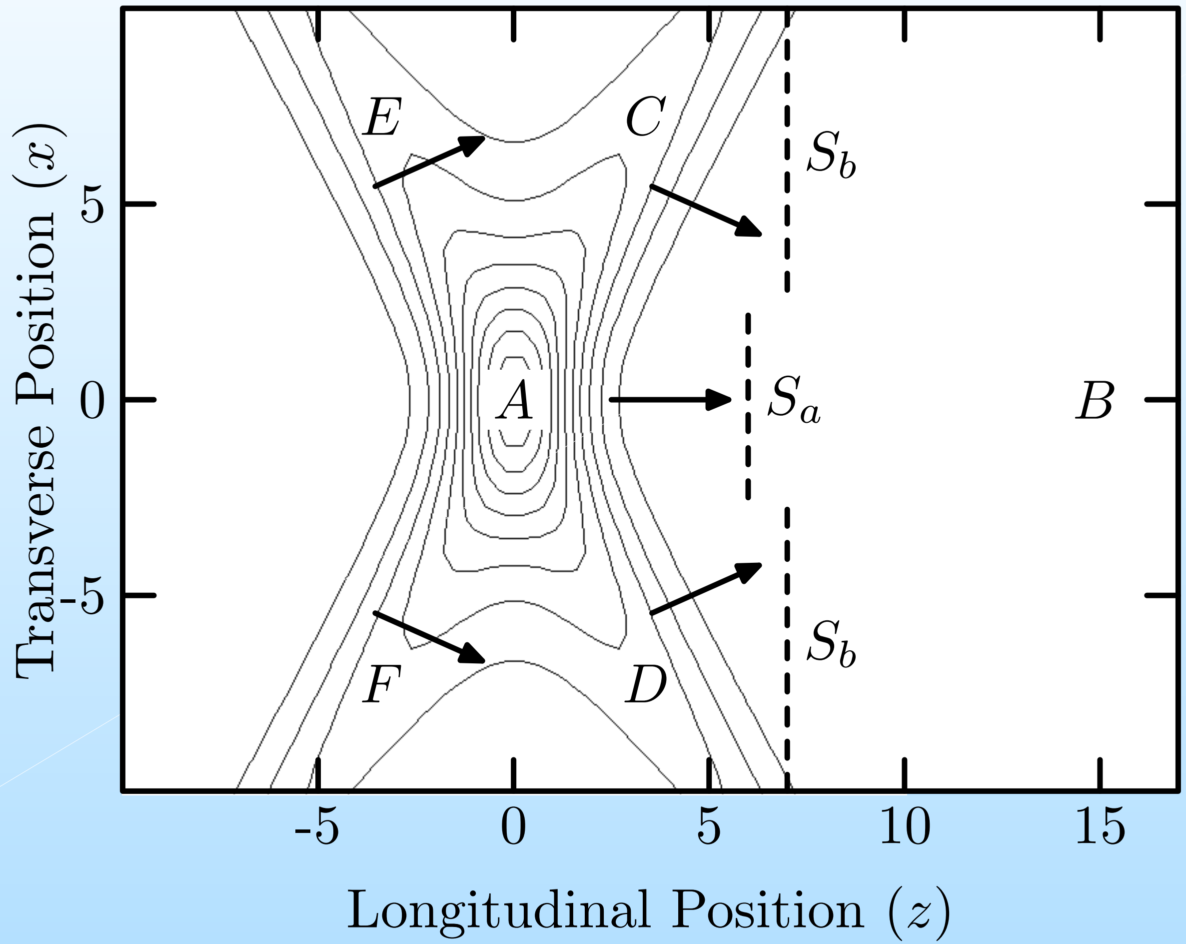
Other *superluminal* systems



X pulses
Bessel pulses

$$\left.\begin{array}{r} X \text{ pulses} \\ \text{Bessel pulses} \end{array}\right\} v_f = v_g = c/cos\theta > c$$

Bessel Pulses



$\theta = 30^\circ$, $\omega_0 = 0$, $\Delta\omega = c/d$.

Conclusions

- Although the equations are similar, FTIR differs from 1D Q.M. tunneling.
- FTIR is a 3D phenomenon, as it requires well defined angles$\Rightarrow$ laterally extended wavefronts.
- Subluminal propagation from lateral wings of incident pulse generates transmitted pulse, yielding the illusion of superluminality.
- Subluminal propagation from centroid of incident pulse contributes only to lateral wing of transmitted pulse.
- Opaque screens may demonstrate subluminal propagation, but they add non-evanescent contributions.
- Propagation of pure evanescent pulses is *indistinguishable* from superluminal acausal propagation.
- Other inhomogeneous waves.