Non Local Effects in the Casimir Force

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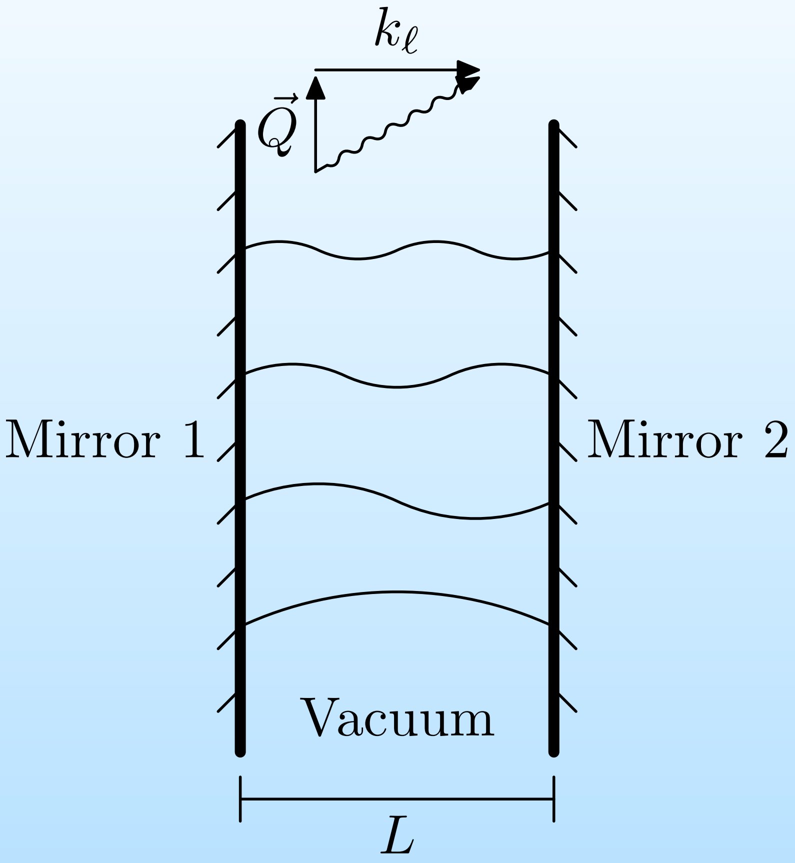
Vacuum Fluctuations



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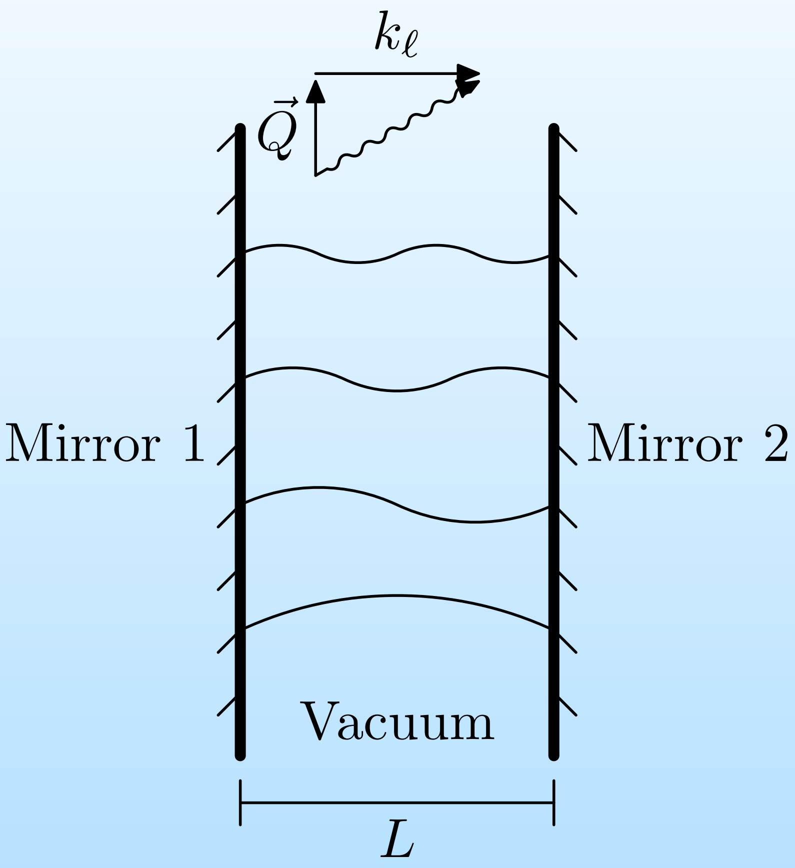

- First quantization:

$$k_\ell = \ell\pi/L$$

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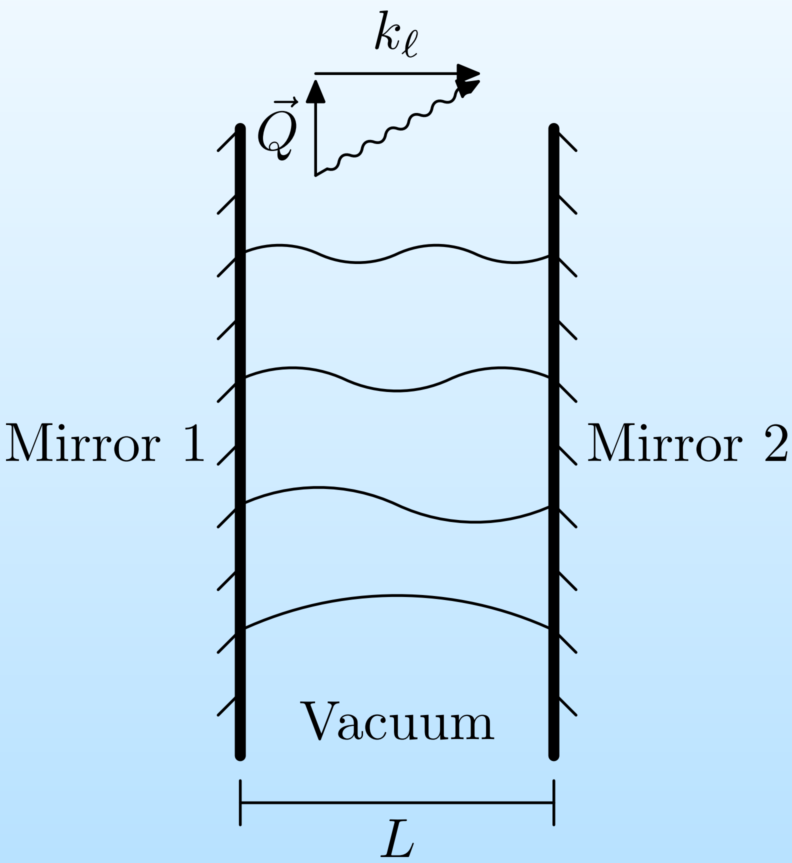

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- Wave eq. $\Rightarrow$ harmonic oscillators:

$$\begin{aligned} \frac{\partial^2}{\partial t^2}\phi_{k_\ell} &= c^2\nabla^2\phi_{k_\ell} \\ &= -k_\ell^2 c^2 \phi_{k_\ell} \end{aligned}$$

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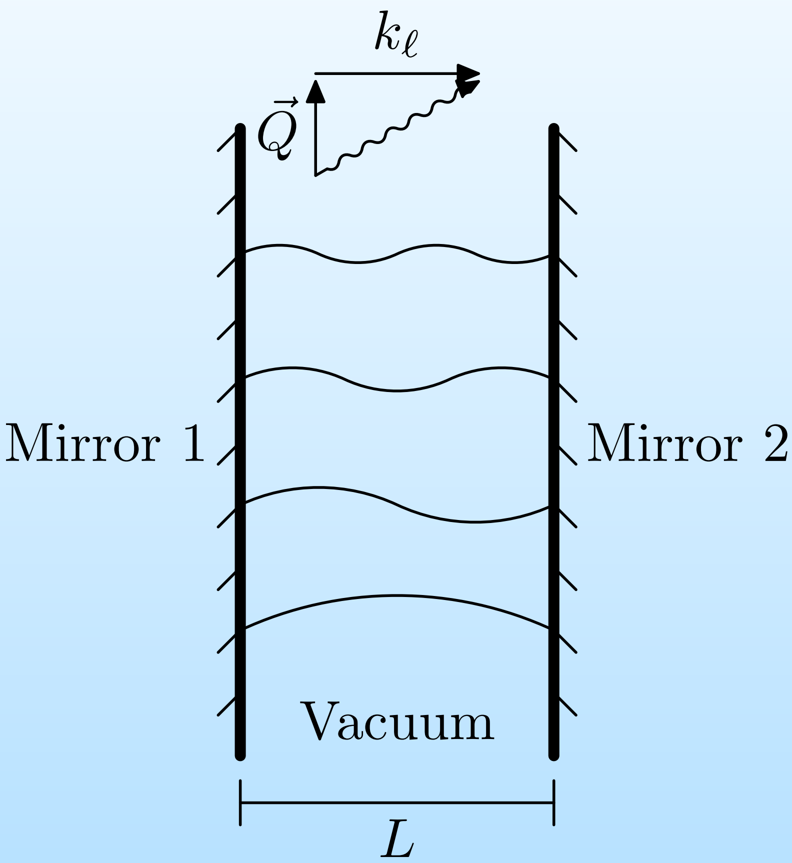

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- $\omega_\ell = k_\ell c = \ell\pi c/L$

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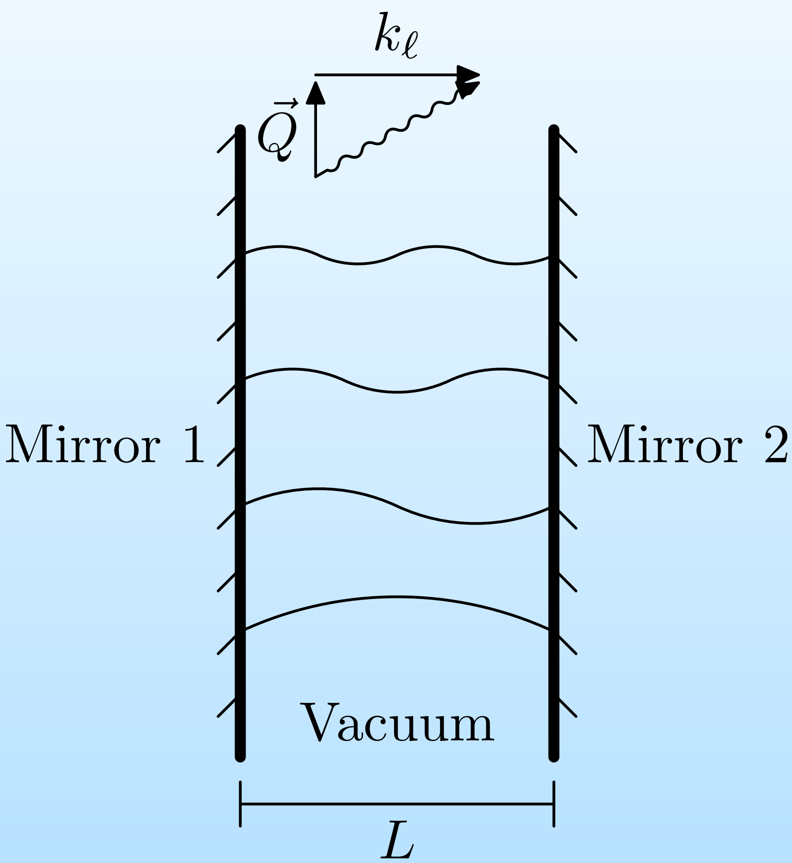

- Second quantization:

$$
\begin{aligned}
E_{n_\ell} &= \left(n_\ell + \frac{1}{2}\right)\hbar\omega_\ell \\
&= \frac{\pi\hbar c}{L}\left(n_\ell + \frac{1}{2}\right)\ell
\end{aligned}
$$

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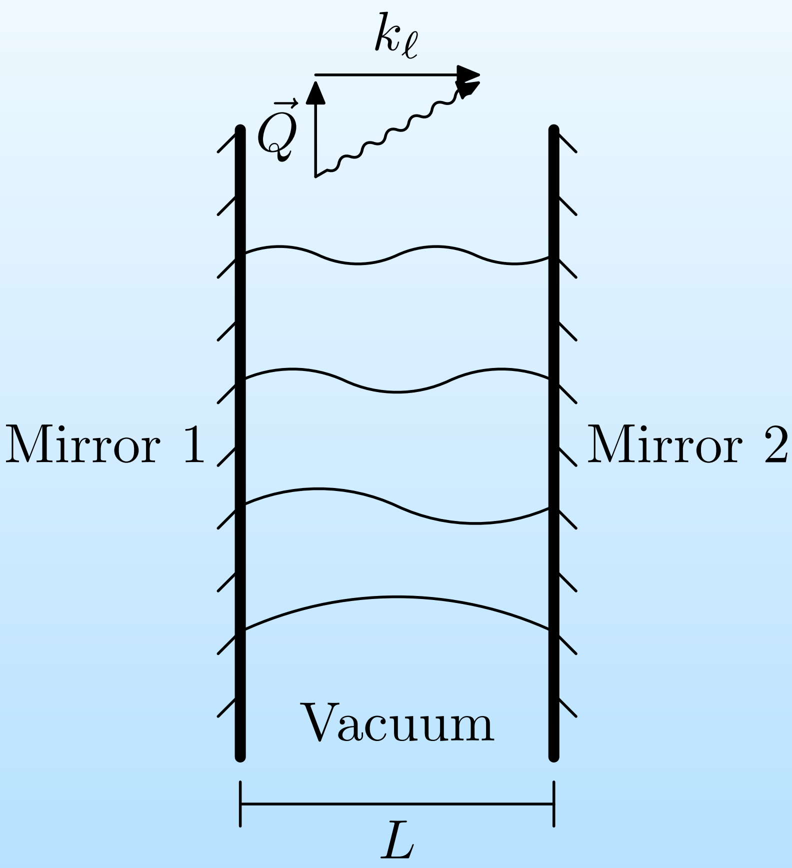

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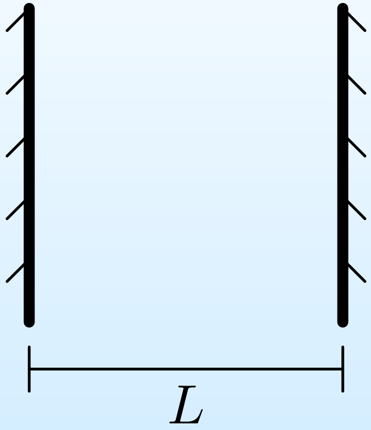
- Vacuum energy:

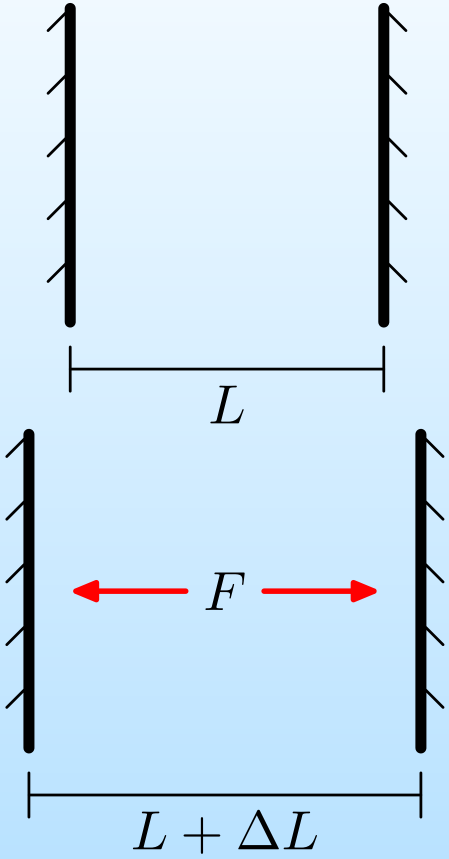
$$U(L) = \sum E_{n_\ell}$$
$$= \frac{\pi\hbar c}{L}\sum\left(n_\ell + \frac{1}{2}\right)\ell$$

Casimir Force



$U(L)$

Casimir Force



$$U(L)$$

$$U(L + \Delta L) = U(L) - F(L)\Delta L$$

Problem

$U(L)$ diverges even at $T = 0$ (*ultraviolet catastrophe*),

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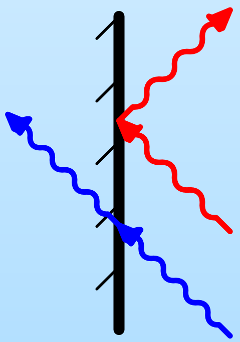
Discussion:

- Perfect mirrors?
- Cutoff frequency



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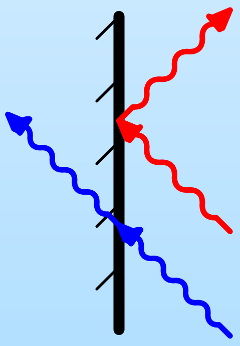

Regularization: Riemann's *zeta*

- $\sum_\ell \ell^s = \zeta(-s)$ (if $s < -1$).
- $U_s(L) = \frac{\pi\hbar c}{2L} \sum l^s$

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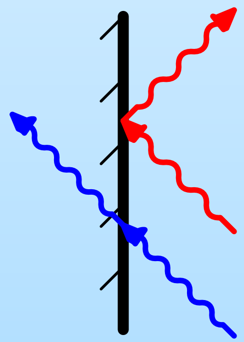

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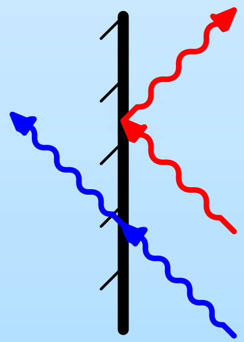

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- $\sum_\ell \ell^s = \zeta(-s)$ (if $s < -1$).
- $U_s(L) = \frac{\pi\hbar c}{2L} \sum l^s$

$$\begin{aligned} U(L) &= \infty + \lim_{s\to -1} U_s(L) \\ &= \infty + \frac{\pi\hbar c}{2L}\zeta(-1) \end{aligned}$$

Casimir Effect

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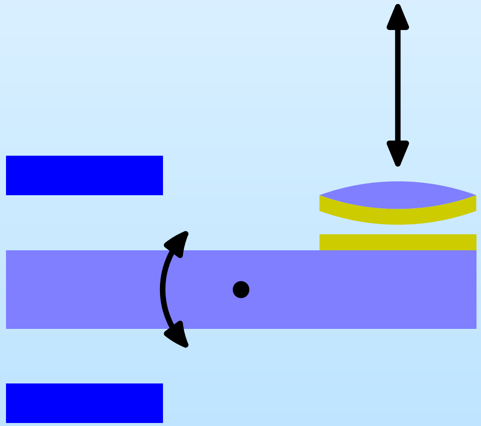
- $F(L) = -\dfrac{dU(L)}{dL} = -\dfrac{\pi \hbar c}{24L^2}$,
- Quantum fluctuations of the electromagnetic field manifest themselves as an attractive force between nearby surfaces.
- In 3D+ two polarizations (TE y TM):

$$F(L) = -\frac{\pi^2 \hbar c \mathcal{A}}{240 L^4}.$$

Resurgence

- *Known since 1948 (Casimir)*.
- Confirmed in 1958 *with 100% uncertainty* (Sparnay).
- Measured again in 1997 with a torsion pendulum (Lamoreaux) with 5% uncertainty and $L \sim 600$nm.



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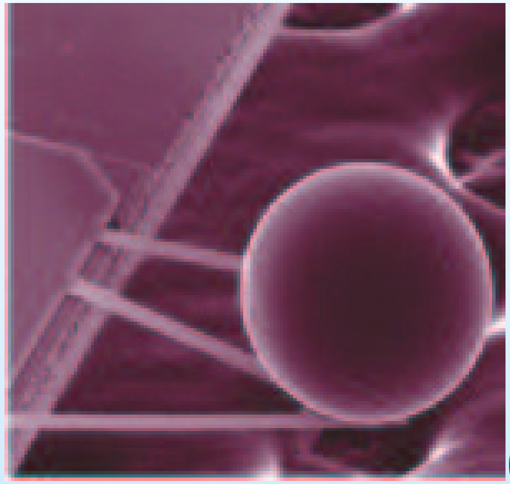
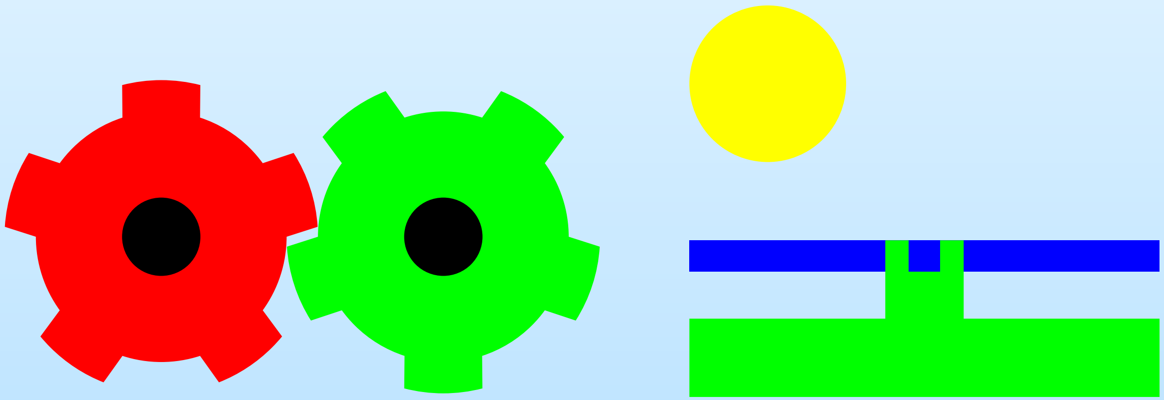

(Mohideen)

- AFM's have allowed $\sim 1\%$ precision down to $L \sim 100$nm. Small but significant deviations.
- Atom manipulation through zero point field. . .
- Casimir and Cosmology: G depends (?) on vacuum fluctuation energy. . .

Nano Machines

- The Casimir force decays very fast with distance $\propto L^{-4}$, but at 10nm it is equivalent to $\sim$1atm! It might play a role in micro and nano machines.



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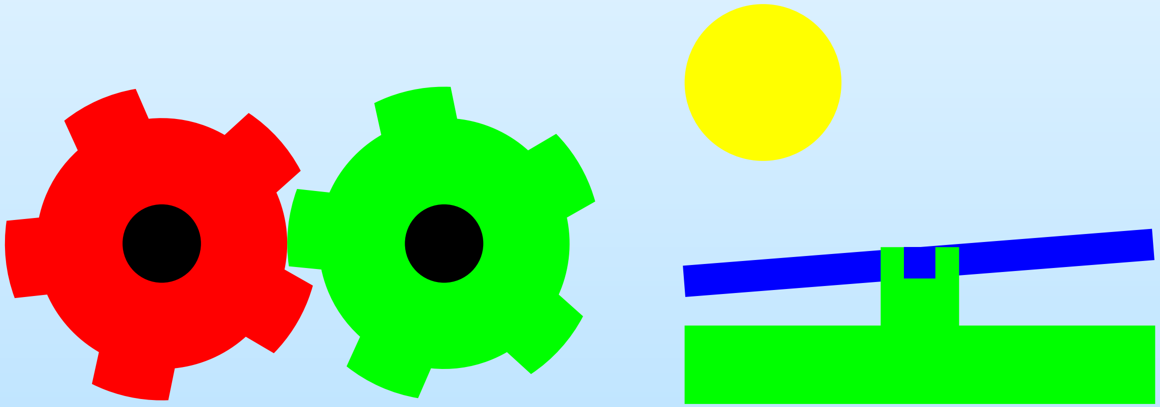

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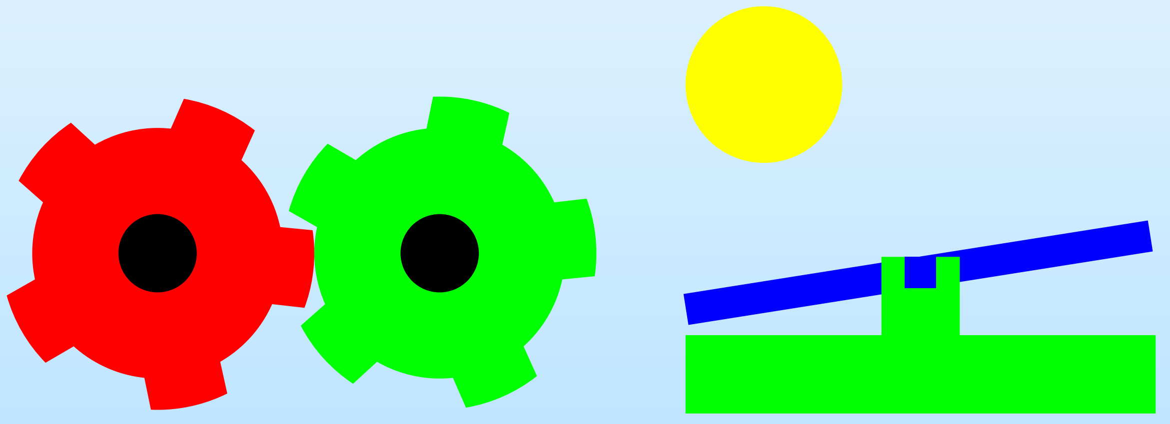

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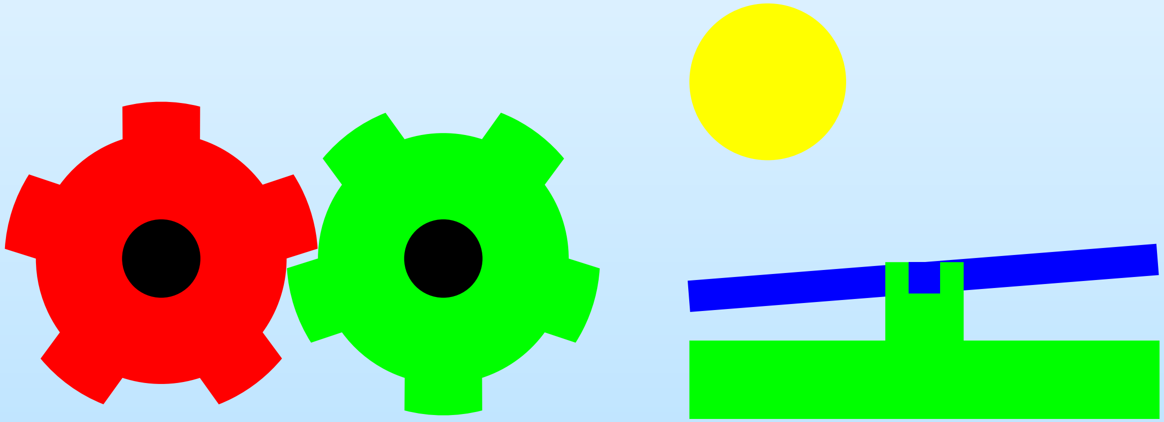

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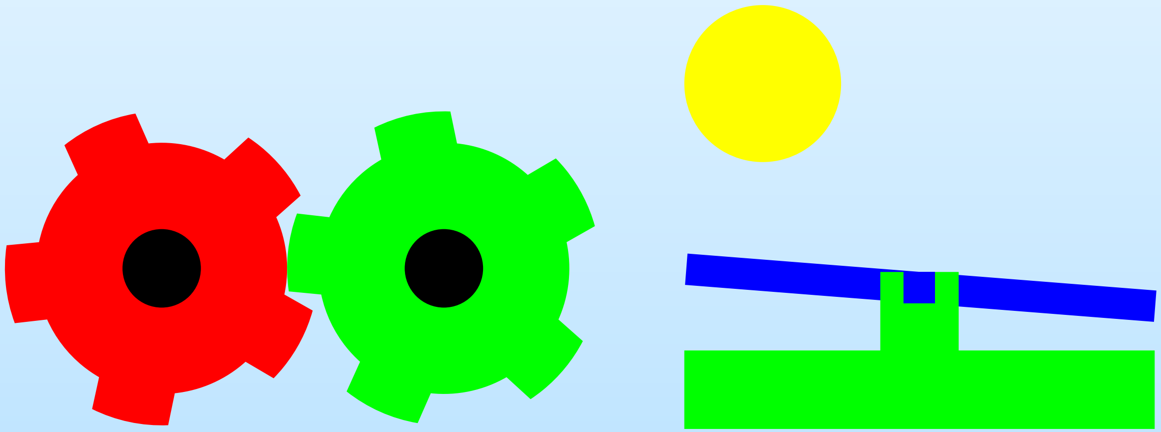

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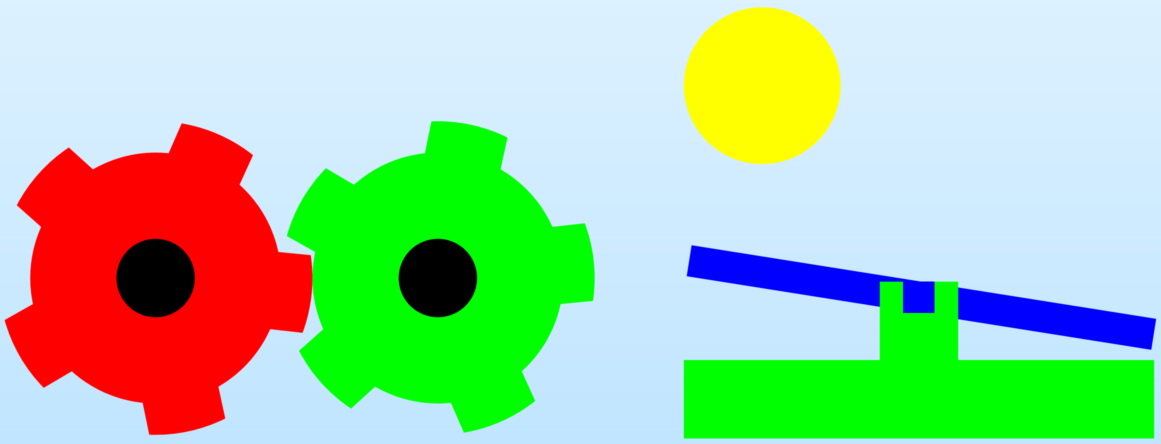
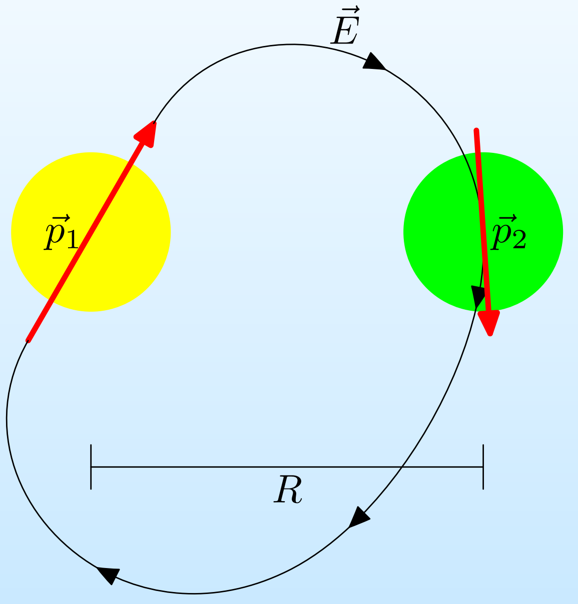


Real Materials



- van der Waals:

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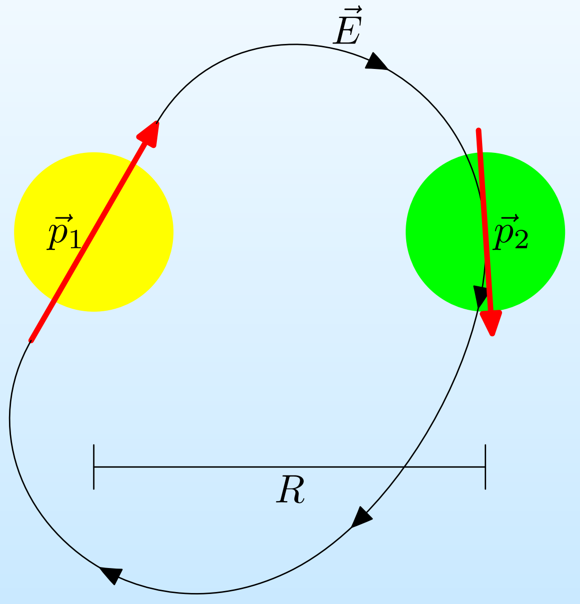

- van der Waals:

  p_1

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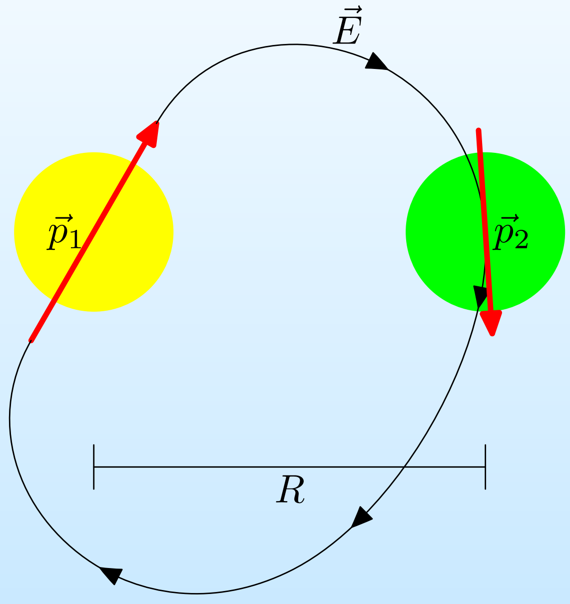

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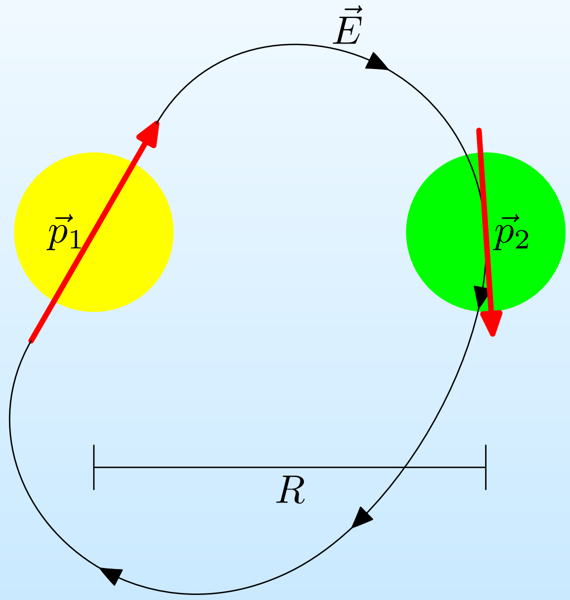

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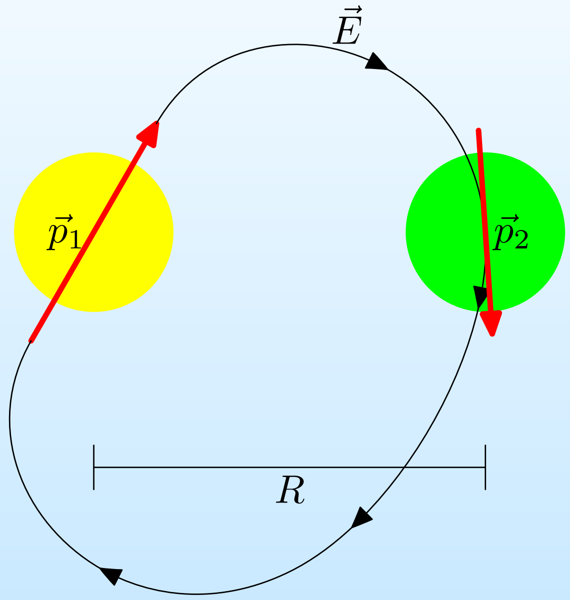

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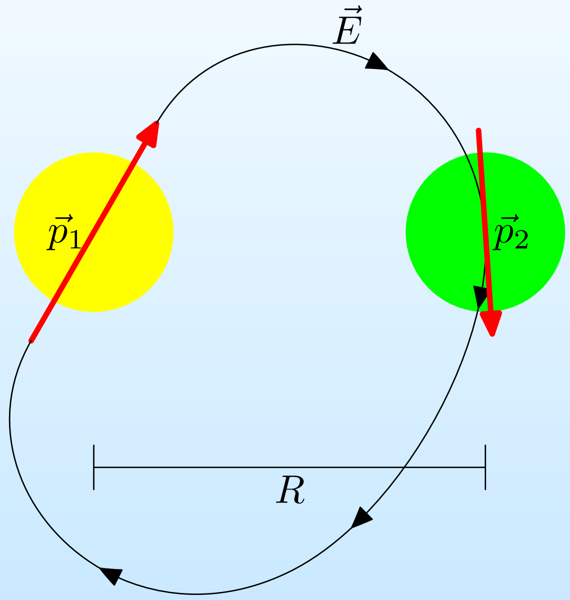

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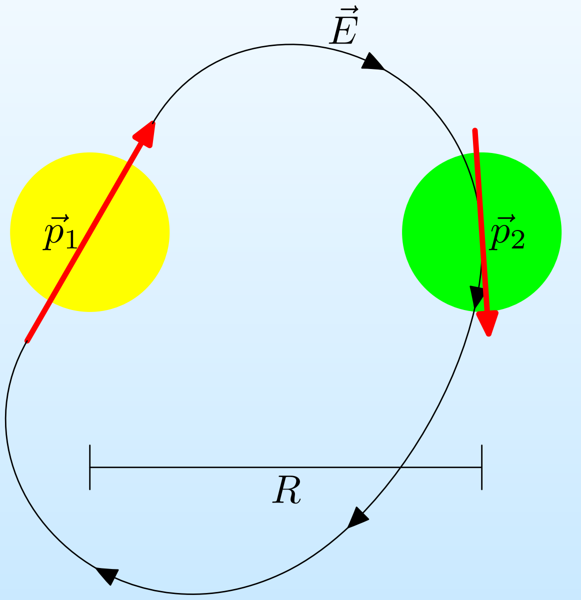

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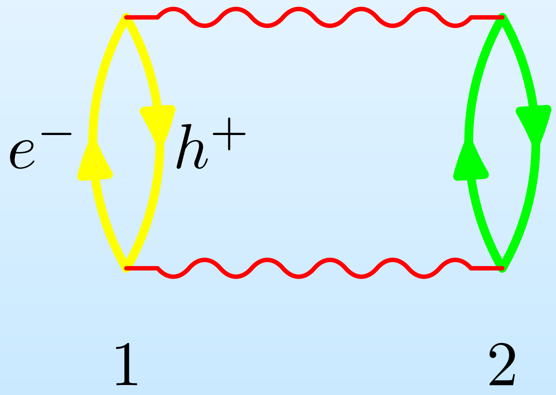

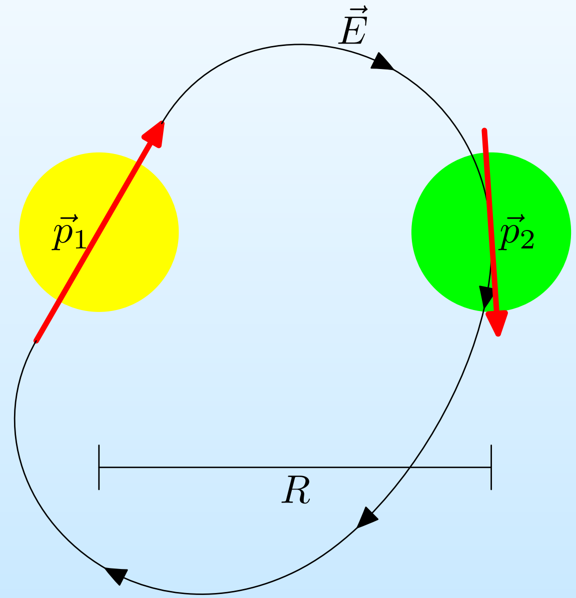

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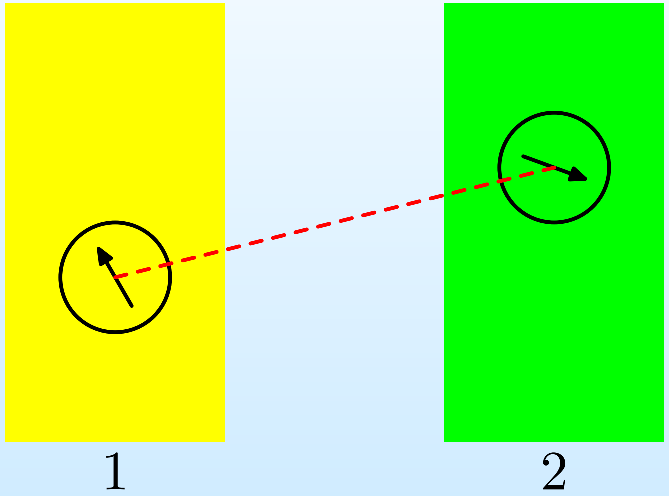
$$U = -\frac{3\hbar}{\pi R^6} \int_0^\infty du\, \alpha_1(iu) \alpha_2(iu)$$

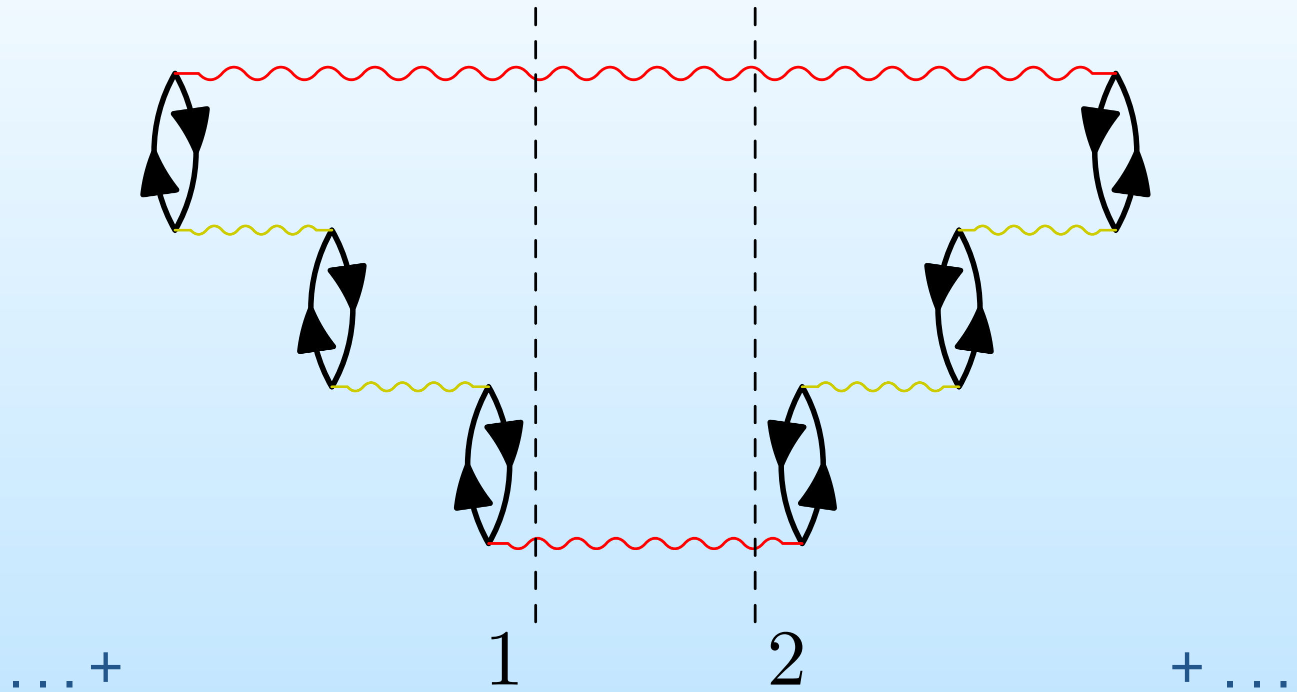
Superposition



$$U = \sum_{ij} \vec{U}_{ij} \propto \int d^3r_1\, d^3r_2\, \frac{1}{R_{12}^6} \propto \frac{\mathcal{A}}{L^2}.$$

- $F \propto L^{-3}$
- Retardation: $R_{ij}^6 \to R_{ij}^7$, $F \to L^{-4}$
- Additivity?
- Geometry and the sign of the force.

Elementary Excitations of a Solid



- An enumeration and a sum over *all* diagrams would require a specific microscopic model for the material.

Dressed Photons

- Within a material $c \to c/\sqrt{\epsilon}$
- $\epsilon = \epsilon(\omega) \to \epsilon_a(\omega)$, $a =$ material or vacuum.
- $\nabla^2 \vec{A} + \epsilon_a(\omega)\frac{\omega^2}{c^2}\vec{A} = 0$ + B.C. $\Rightarrow$ normal electromagnetic modes. From normal modes $\Rightarrow$ energy and force.
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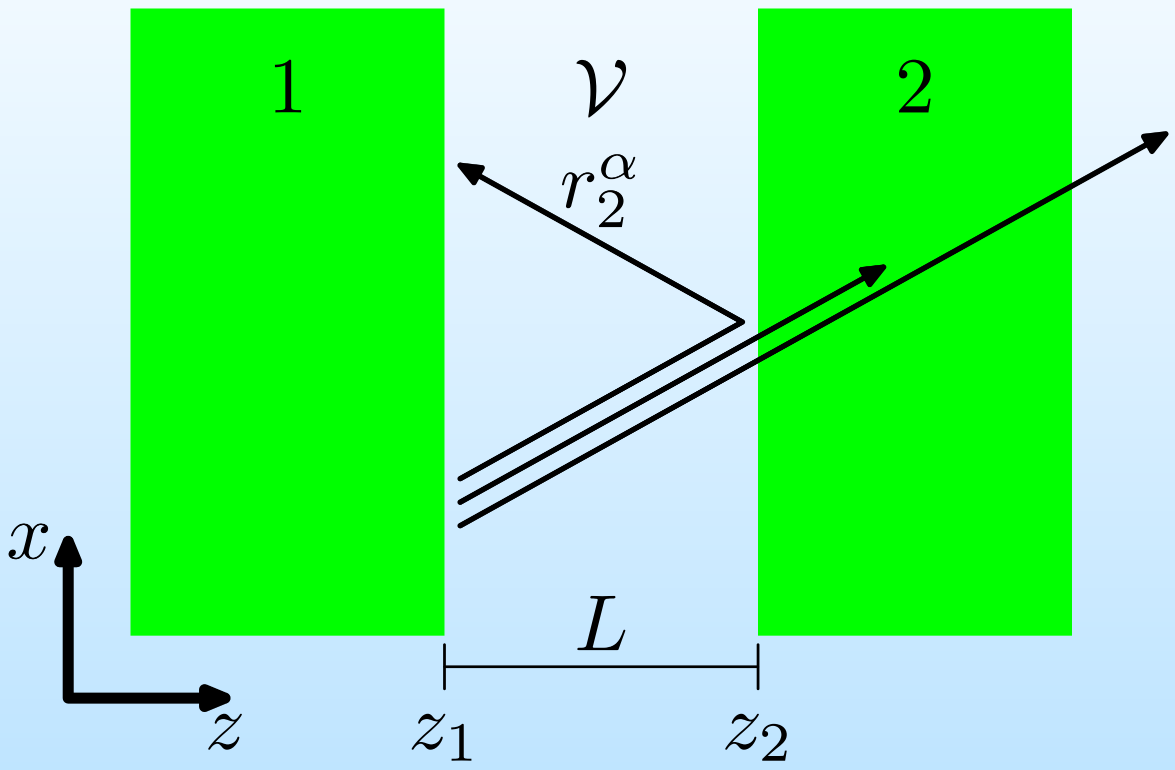
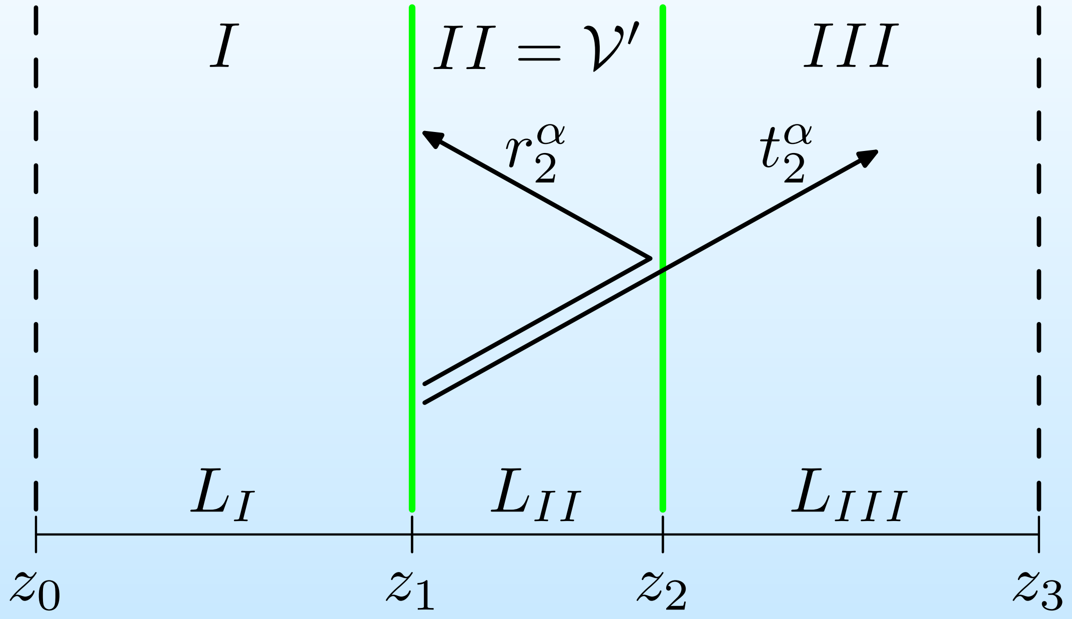
  But...
- $\epsilon_a(\omega)$ are complex, i.e., they display *temporal* dispersion and dissipation. Electromagnetic *modes* do not form a complete orthogonal *basis*.
- Fluctuating sources $\vec{j}(\vec{r}, t)$. $\langle j_i \rangle = 0$, but $\langle j_i j_j \rangle \neq 0$.
- Hidden assumptions: homogeneity, isotropy, locality...

General Derivation



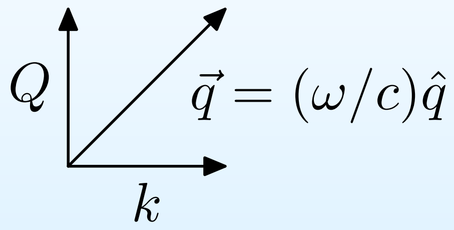
- Real system:
- Detailed balance:
  Coherent reflection r_2^α. Incoherent emission $1 - |r_2^\alpha|^2$.
  Within the cavity *everything depends on* r_a^α *exclusively!*

Fictitious System



- Choose r_a^α as in the real system,
- Choose t_a^α to conserve energy. *No absorption!*
- $L_{II} \ll L_I, L_{III} \to \infty$.
- Perfect mirrors at z_0, z_3 to quantize and count modes. . . .

EM Modes



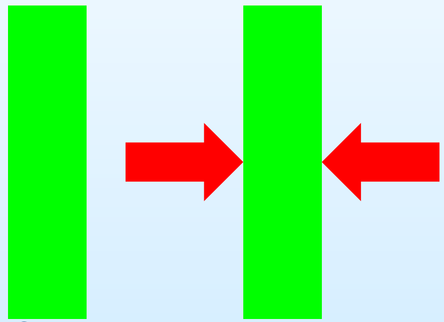
- s pol.: $\vec{E} = (0, E_y(z), 0)e^{i(Qx-\omega t)}$.
- $\left(\frac{d^2}{dz^2} + k^2\right) E_y = 0$, $k^2 = \omega^2/c^2 - Q^2$
- Apply B.C. at $z_1 = 0$, $z_2 = L$ (and at z_0 and z_3).
- Obtain stress tensor within cavity for each mode.
- Relate to energy of mode, i.e., to frequency and occupation number.
- Sum over modes to obtain total momentum flux.

Green's Function

- $E_y^>(z) = e^{i\tilde{k}(z-L)} + r_2^s e^{-i\tilde{k}(z-L)}$ obeys B.C. at right side,
  $E_y^<(z) = e^{-i\tilde{k}z} + r_1^s e^{i\tilde{k}z}$ obyes B.C.at left side.
- *Electric* Green's function: $G^E_{\tilde{k}^2}(z, z') = \dfrac{E_y^<(z_<)E_y^>(z_>)}{W}$.
- *Magnetic* Green's function: $E_y \rightarrow B_x$, $r_a^s \rightarrow -r_a^s$.
- Local density of states:

$$\begin{aligned}\rho^s_{k^2}(z) &= -\frac{1}{2\pi}\text{Im}[G^E_{\tilde{k}^2}(z,z) + G^B_{\tilde{k}^2}(z,z)] \\ &= \frac{1}{2\pi\tilde{k}}\text{Re}\left(\frac{1 + r_1^s r_2^s e^{2i\tilde{k}L}}{1 - r_1^s r_2^s e^{2i\tilde{k}L}}\right).\end{aligned}$$

Momentum Flux



For each photon:

- Momentum $p_z = \pm\hbar k$,
- Velocity $v_z = \pm ck/q$,
- Contribution to momentum flux $-t_{zz} = +\hbar ck^2/q$,

Adding $tzz\rho_{k^2}$ over all modes, using $\sum_{k^2} \to \int kdk$, $\sum_{\vec{Q}} \to \mathcal{A}/(4\pi)\int QdQ$, and $\alpha = s, p$ we obtain the stress tensor T_{zz} within the cavity. Substracting the stress tensor on the outside we obtain the force on a slab…

Lifshitz Formula

$$\frac{F}{\mathcal{A}} = \frac{\hbar c}{2\pi^2}\int_0^\infty Q dQ \int_{q\geq 0} dk\, \frac{k^3}{q} f\, \text{Re} \frac{1}{\tilde{k}}\left(\frac{1}{\xi^s - 1} + \frac{1}{\xi^p - 1}\right).$$

- $f = N + 1/2 =$occupation number of state $\vec{Q}, k, \alpha$,
- $\xi^\alpha = (r_1^\alpha r_2^\alpha e^{2i\hat{k}L})^{-1}$.

Unlike Lifshitz' and other's derivations, we made no assumptions about the slabs except symmetry along $x-y$ and isotropy around z; they may be semiinfinite, finite or thin films; homogeneous, inhomogeneous, layered, ordered, or disordered; transparent or absorptive; conducting or insulating; *local or non local*. . .

Spatial Dispersion

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- Similarly, the response at $\vec{r}$ might depend on the excitation at $\vec{r}' \neq \vec{r}$,

$$D_i(\vec{r}, t) = \int d^3r' \int dt' \, \epsilon_{ij}(\vec{r}, \vec{r}'; t - t')E_j(\vec{r}', t').$$

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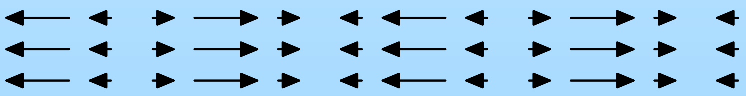
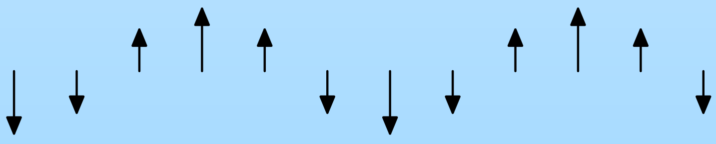
$$D_i(\vec{r}, t) = \int d^3r' \int dt' \, \epsilon_{ij}(\vec{r}, \vec{r}'; t - t') E_j(\vec{r}', t').$$

- For an isotropic, homogeneous medium,

$$\vec{D}^L(\vec{q}, \omega) = \epsilon^L(q, \omega) \vec{E}^L(\vec{q}, \omega), \qquad \vec{D}^T(\vec{q}, \omega) = \epsilon^T(q, \omega) \vec{E}^T(\vec{q}, \omega).$$

Hydrodynamic Model

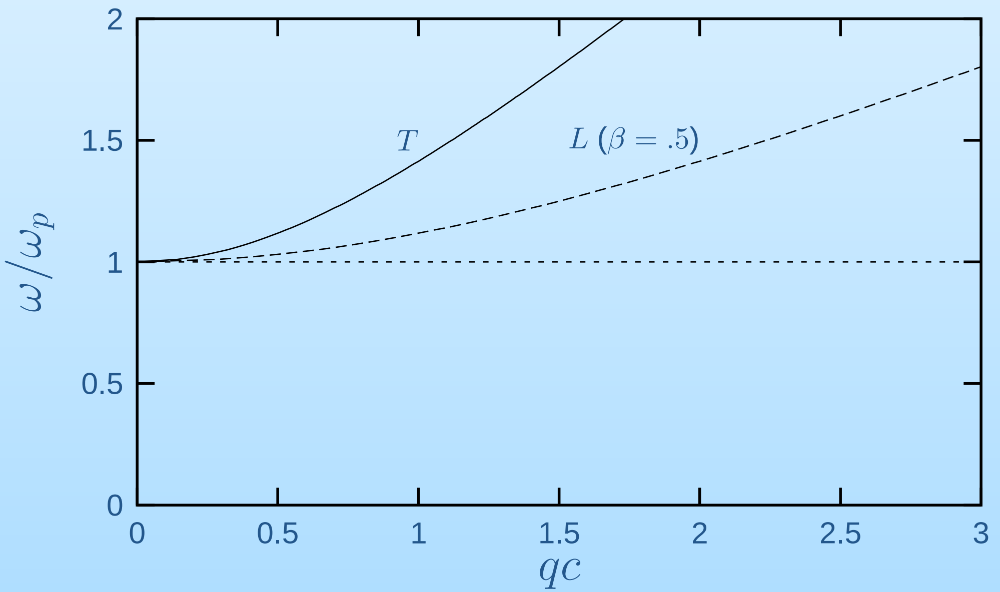
Semiclassical compressible fermion gas

- Longitudinal wave: $n \to n + \delta n$, $\delta n \propto \nabla \cdot \vec{P}$.
- Energy: $\delta U \propto \delta n$.
- Presión: $\mathcal{P} \propto \partial U / \partial n$.
- Fuerza: $\vec{f} \propto -\nabla \mathcal{P} \propto \nabla \delta n \propto \nabla \nabla \cdot \vec{P} = -q^2 \vec{P}^L$.
- $-\omega^2 \vec{P}^L \propto \ldots - q^2 \vec{P}^L$.

- $\epsilon^T(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2 + i\omega/\tau}$.
- $$\epsilon^L(\vec{q}, \omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega/\tau - \beta^2 q^2}.$$
- $\omega_p^2 = 4\pi n e^2 / m$.
- $\beta^2 = v_F^2/3 \to 3v_F^2/5$.

Consecuencias

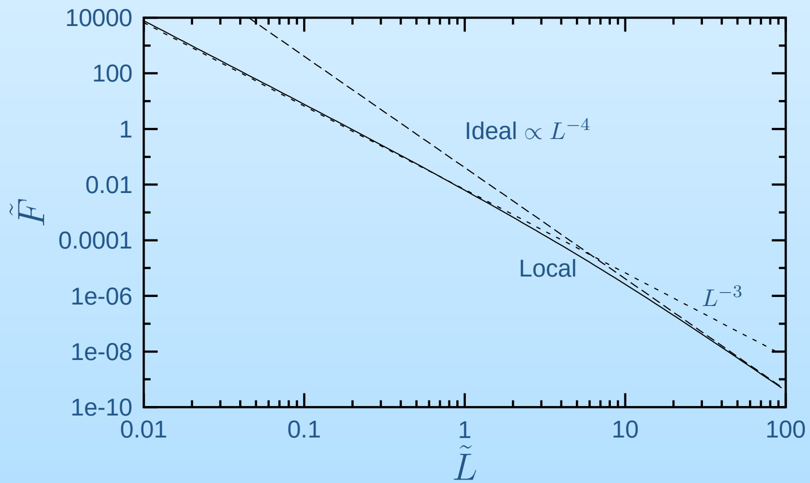
- Ondas transversales: $q^2 = \epsilon^T(w)\dfrac{\omega^2}{c^2}$.
- Ondas longitudinales: $\nabla \cdot \vec{D} = 0 \Rightarrow \epsilon(\vec{q}, \omega) = 0$, $q^2 = (\omega^2 - \omega_p^2)/\beta^2$.

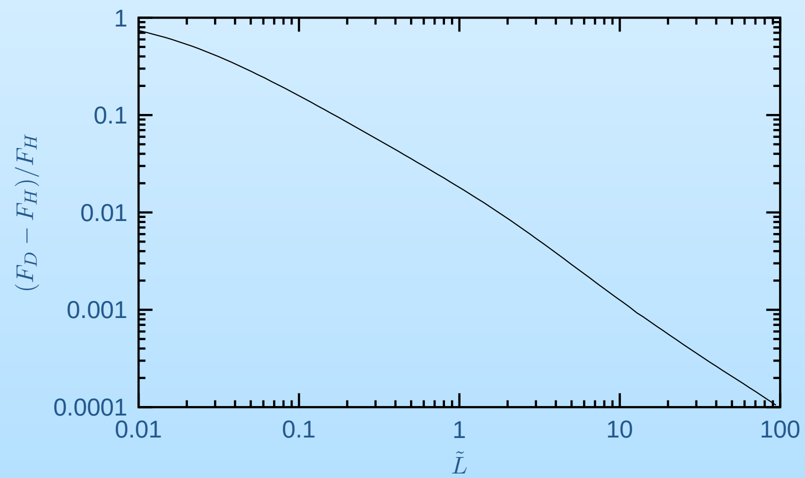


- Apantallamiento: $\kappa_{TF} = \omega_p/\beta$
- ABC's $\Rightarrow r_s$, r_p.

Resultados (1)

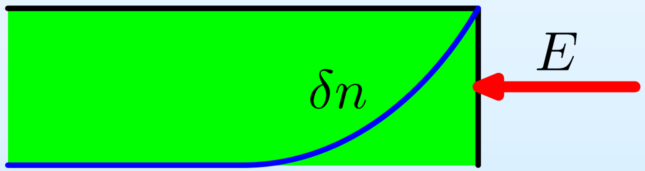
- Parámetros ajustados a Au.
- $\tilde{L} = 2\pi L/\lambda_p$.
- $\tilde{F} = (\lambda_p/2\pi)^4 F/\mathcal{A}\hbar c$.
- $\lambda_p = 2\pi c/\omega_p$.



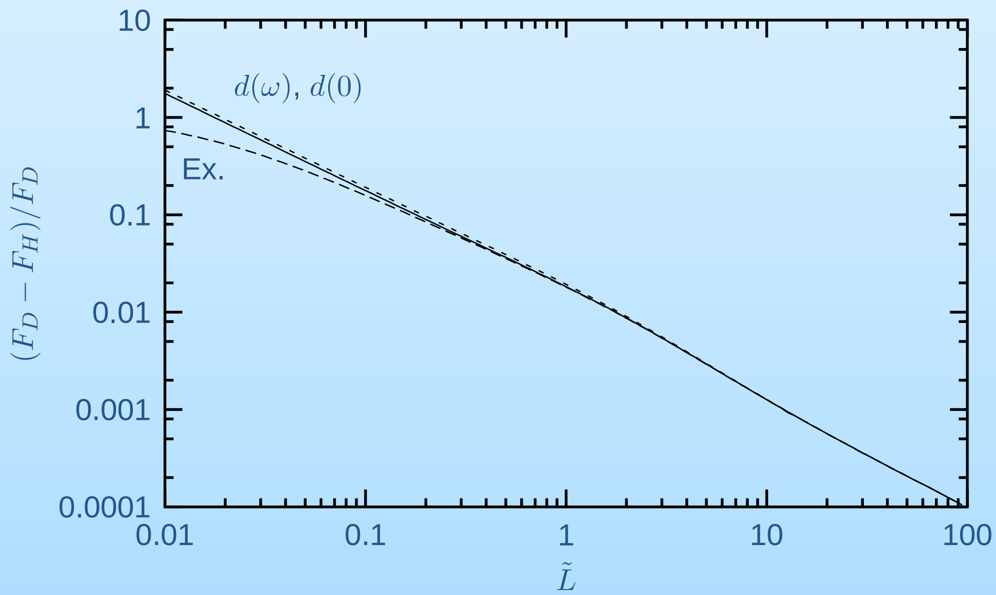


Parámetros d

- La no-localidad disminuye F. Generación de plasmones. . . o



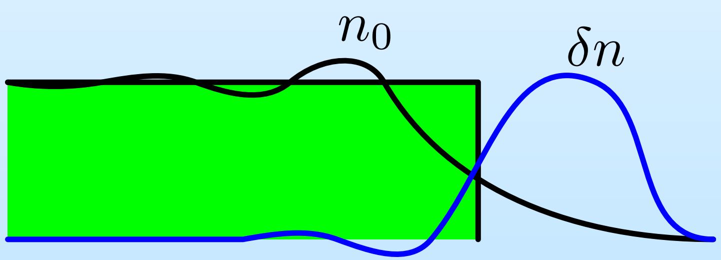
- $L_{ef} > L$, $F_H < F_D$.
- $d \equiv \dfrac{\int dz\, z\delta n}{\int \delta n}$.
- $r^p \approx r^p_D + (\ldots)d$.



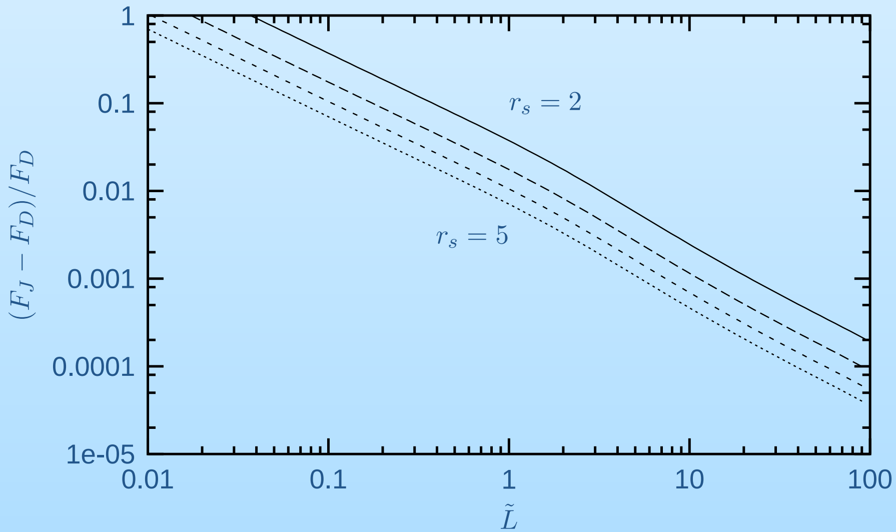
Discusión

- La corrección no local puede ser cercana al 100%.
- d funciona para $\tilde{L} > 0.1$
- $d(0)$ y $d(\omega)$ $\Rightarrow$ resultados similares.

Jellium



- El centroide de carga se desplaza hacia el vacío.
- La corrección no-local cambia de signo.



Conclusiones

- Deducción de la fórmula de Lifshitz que permite calcular la fuerza de Casimir entre materiales *arbitrarios*.
- Sistema ficticio *no-disipativo*, sin grados de libertad materiales.
- El único ingrediente del cálculo es la amplitud de reflexión de cada superficie.
- Modelo hidrodinámico simple $\Rightarrow$ cálculo exacto. La fuerza de Casimir se reduce significativamente por los efectos no locales.
- Interpretación en términos de d.
- Los valores estáticos de d dan buenos resultados. . .
- Calculos de *jellium* autoconsistente $\Rightarrow$ d tiene el signo contrario $\Rightarrow$ la corrección no local cambia de signo y la fuerza de Casimir aumenta.